VIBRATIONAL ANALYSIS OF DELAMINATED PLATES

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

BACHELOR OF TECHNOLOGY
IN
MECHANICAL ENGINEERING

BY

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AND

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Department Of Mechanical Engineering
National Institute of Technology
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CERTIFICATE

This is to certify that the thesis entitled, **"Vibration Analysis of Delaminated Plates"** submitted by **MR. KUMAR AMAN** and **MR. KULWANT SINGH PARIHAR** in partial fulfillment of the requirements for the award of Bachelor of Technology degree in Mechanical Engineering at National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance. To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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CONTENTS

ABSTRACT

The delamination phenomenon is common in composite beams as the composite beams are having laminate structures. Delamination leads to development of cracks which reduces the strength of the material and ultimately the material fails to bear the desirable load. In this project, the effect of delamination on free vibration of a rectangular plate with through width delamination was investigated using a finite strip method. The basic understanding of the influence of delamination on natural frequencies of delaminated plate is presented using Ansys13.0. Hamilton's principle is used to derive the equations of motion. In addition other factors affecting the vibration of delaminated plates are discussed. The variables of delamination are:

1. Location of delamination
2. Size of delamination
3. Mode of frequency

The numerical results for free vibration of delaminated plates are presented. As expected, the natural frequency decreases with increase in delamination length. These results obtained from ANSYS 13.0 are compared with the results of other case studies. The simulation and graphs are plotted to correlate the natural frequency and delamination variables.

CHAPTER~1

1.INTRODUCTION

1.1 Delamination

Delaminations are cracks inside the interior of the laminate. It is also called barely visible impact damage (BVID)[1], which is not readily identified by visual inspection. Delaminations are commonly found in laminated structures as they are made up in the form of laminate.Delaminations are caused by shocks, impact loading or repeated cyclic stresses which causes a degradation of overall stiffness and strength of the material. Delamination may also develop due to manufacturing defects such as incomplete wetting and entrapped air bubbles between layers. They may also develop as a result of certain in service factors, such as low velocity impact by foreign objects, for instance, dropped tools or bird strikes [2]. Delamination also affects the frequency of the laminated plates; due to delamination it exhibits new vibration modes and frequencies which are dependent on size and location of delamination. Using this method if we have knowledge about the natural frequencies and mode shapes of plate containing delamination, we can find the size and location of delamination. Delamination failure may also be detected in the material by its sound; solid composite has bright sound, while delaminated part sounds dull. Other non-destructive testing methods are also used which testing with ultrasound, radiographic imagining and infrared imaging, frequency measurements etc.

1.2 Objective and Scope of work

In this project, we are using Finite Strip Method (as described by Shiau and Zeng [7] in their case study) to formulate the equations of motion of a rectangular homogeneous plate with through width delamination. The variables of delamination are location of delamination, size of delamination and mode of frequency. The natural frequency of the homogeneous rectangular plate will be found out at different variables of delamination using Ansys13.0. The results will be compared with the results found by finite strip method. Using these results, frequency and delamination variables will be correlated.

CHAPTER~2

2. LITERATURE SURVEY

Jun et al.[3] introduced a dynamic finite element technique for free vibration analysis of typically laminated composite beams on the idea of 1st order shear deformation theory. The influences of Poisson impact, couplings among extensional, bending and torsional deformations, shear deformation and rotary inertia are incorporated within the formulation. The dynamic stiffness matrix is formulated primarily based on the precise solutions of the differential equations of motion governing the free vibration of generally laminated composite beam. The effects of Poisson effect, material anisotropy, slender ratio, shear deformation and boundary condition on the natural frequencies of the composite beams are studied thoroughly. The numerical results of natural frequencies and mode shapes are presented and, whenever possible, compared to those previously published solutions so as to demonstrate the correctness and accuracy of the current technique.

Hu et al.[4] proposed a FEM model for vibration analysis of delaminated composite beams and plates based on a simple higher-order plate theory, which can satisfy the zero transverse shear strain condition on the top and bottom surfaces of plates. To set up a C^0-type FEM model, two artificial variables have been introduced in the displacement field to avoid the higher-order derivatives in the higher-order plate theory. The corresponding constraint conditions from the two artificial variables have been enforced effectively through the penalty function method using the reduced integration scheme within the element area. Furthermore, the implementation of displacement continuity conditions at the delamination front has been described using the present FEM theory.

Lee[5] proposed a layerwise model to formulate the equations of motion of a delaminated plate. Numerical results are obtained and compared with those of other theories addressing the effects of the lamination angle, location, size and number of delamination on vibration frequencies of delaminated beams. It is found that a layerwise approach is adequate for vibration analysis of delaminated composites

Thambiratnam et al. [6] have implemented finite element technique to review the free vibration analysis of isotropic beams with uniform cross section on an elastic foundation using Euler-Bernoulli beam theory

Shiau et al.[7] introduced finite strip method to investigate the effect of delamination on free vibration of a simply supported rectangular homogeneous plate with through-width delamination. A constrained model was used and a finite strip with bending and in-plane stiffness was derived for the free vibration analysis. The effects of delamination length, delamination location in the thickness-wise and span-wise directions, and aspect ratio of the plate on the natural frequencies of the plate were presented.

CHAPTER~3

3. Numerical modeling and formulation

3.1 Formulation:

In the present analysis we are using a finite element method for free vibration analysis of delaminated plates. We will consider a rectangular plate with through width delamination.

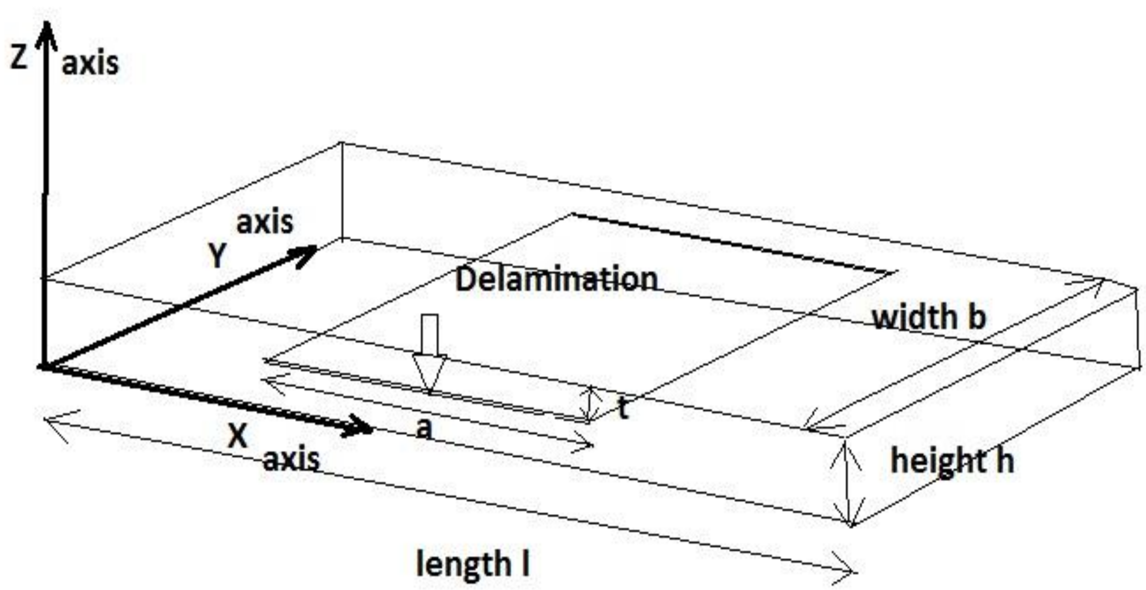


Diagram: Rectangular plate with through width delamination

Length = l, Width =b, Height=h

X and Y are the axis directions in the plane of the plate and Z is perpendicular to this plane. *u,v* are the displacements in X and Y-directionsand *w* is displacement in Z-direction(upward positive).

Assumptions:

Suppose delamination is located at a distance < t > from the top surface and the length of delamination is < a > and delamination is centrally located.

From the theory of bending homogeneous plates we have the basic idea about the constitutive equation of a homogeneous thin plate. But if we will also consider the in-plane forces in addition to bending moments and different stiffness of each lamina, it will produce a different constitutive equation with elements of coupling. Here we are just giving some description about this approach to derive constitutive equation of a homogeneous plate (with no coupling effect).

1. {F} and {M} are force and moments applied to the plate at a position (x,y).
2. $\{\in^0\}$ is the mid plane strain and $\{\mathrm{k}\}$ is the curvature (second derivatives of the displacement)

$$\mathrm{F} = \begin{Bmatrix} F_x \\ F_y \\ F_{xy} \end{Bmatrix} \ , \mathrm{M} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

$$\{\in^0\}^{\mathrm{T}}=\{\in^0{}_x \quad \in^0{}_y \quad \in^0{}_{xy} \ \} =\left\{\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\} \tag{1}$$

$$\{k\}^{\mathrm{T}}=\{k_x \ k_y \ k_{xy} \ \}=\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \quad 2 \ \frac{\partial^2 w}{\partial xy}\right\} \tag{2}$$

Where; u, v and w are mid-plane displacements and μ is Poisson's ratio.

If no. of laminas considered in given plate is N then

Force equation for N laminas:

$$F = \int_{-h/2}^{+h/2} \sigma \, dz = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \sigma_k \, \mathrm{dz}$$

Similarly moment equation:

$$M = \int_{-h/2}^{+h/2} \sigma . z \, dz$$

After solving the integrals for F and M, they can be expressed in compact form,

$$F = A\in^{\mathbf{0}} + Bk$$

$$M = B\in^{\mathbf{0}} + Dk$$

These two relations between applied forces and moments, and the resulting mid-plane strain and curvatures,can be summarized in form of a single matrix equation:

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \in^0 \\ k \end{Bmatrix}$$

3. The A/B/B/D matrix in brackets is the laminate stiffness matrix, and its inverse will be the laminate compliance matrix.
4. [A] is an “extensional stiffness matrix", it gives the influence of an extensional mid-plane strain $\in^0$on the in-plane forces F.
5. [B] is “coupling stiffness matrix”, it contributes in the curvature part k of the in-plane force F.
6. [D] is “Bending stiffness matrix”, it contributes in the curvature part k of the moment M.

The presence of nonzero elements in the coupling matrix B is indicating that the application of an in-plane force will lead to a curvature or warping of the plate (coupling effect), or that an applied bending moment M will also generate an extensional strain$\in^0$. These types of effect are not desirable.

Now if we are considering homogeneous plates with no allowance for in-plane forces in addition to bending moment and stiffness characteristics are taken same throughout the plate.

For a homogeneous simply supported plate we are not considering any coupling or warping effect so in [A/B/B/D] matrix coupling part will be zero.

So constitutive equation will become:

$$\begin{Bmatrix} F \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \in^0 \\ k \end{Bmatrix}$$

Here extensional stiffness matrix

$$[A] = \frac{Eh}{(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{(1-\mu)}{2} \end{bmatrix} \quad (3)$$

Bending or flexural stiffness matrix

$$[D] = \frac{Eh^3}{12(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{(1-\mu)}{2} \end{bmatrix} \quad (4)$$

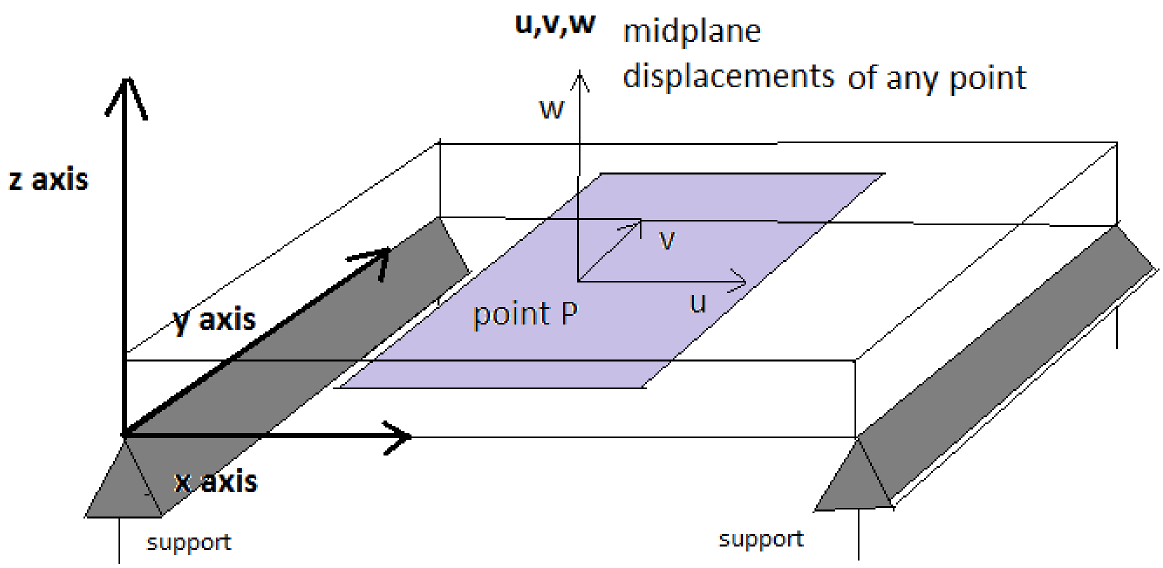


Diagram:simply supported homogeneous delaminated rectangular plate

We are considering a simply supported homogeneous delaminated rectangular plate. The equation of motion for this plate will be found using Hamilton's principle

$$\delta \int_{t1}^{t2} (T - U) dt = 0 \tag{5}$$

Where T= total kinetic energy of the system;

U= total strain energy of the plate; and t= the time of the motion.

The kinetic energy of the plate T, due to the vibration will be:

$$T = \frac{1}{2} \int \rho \dot{w}^2 \, d(vol.) \tag{6}$$

Where w = normal displacement of the plate and $\dot{w}$= derivative of w with respect to time

Strain energy

$$U = \frac{1}{2} \int \{\in^0\}^{\mathrm{T}} [\mathrm{A}] \{\in^0\} \, d(vol.) + \frac{1}{2} \int \{k\}^{\mathrm{T}} [\mathrm{D}] \{k\} \, d(vol.) \tag{7}$$

$$U = \frac{1}{2} \int \left\{ \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\} . \frac{Eh}{(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{(1-\mu)}{2} \end{bmatrix} . \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} d(vol.)$$

$$+ \frac{1}{2} \int \left\{ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \quad 2 \, \frac{\partial^2 w}{\partial xy} \right\} . \frac{Eh^3}{12(1-\mu^2)} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{(1-\mu)}{2} \end{bmatrix} . \begin{Bmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \, \frac{\partial^2 w}{\partial xy} \end{Bmatrix} d(vol.)$$

$$U = \frac{1}{2} \frac{Eh}{(1-\mu^2)} \iiint \left\{ \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\} . \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{(1-\mu)}{2} \end{bmatrix} . \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} dx.dy.dz$$

$$+\frac{Eh^3}{24(1-\mu^2)}\iiint \left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \quad 2\,\frac{\partial^2 w}{\partial xy}\right\}.\begin{bmatrix}1 & \mu & 0\\ \mu & 1 & 0\\ 0 & 0 & \frac{(1-\mu)}{2}\end{bmatrix}.\left\{\begin{matrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\,\frac{\partial^2 w}{\partial xy}\end{matrix}\right\} dx.dy.dz$$

$$U=\frac{1}{2}\frac{Eh}{(1-\mu^2)}\iiint \left\{\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y} \quad \mu\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\frac{(1-\mu)}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\}.\left\{\begin{matrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{matrix}\right\} dx.dy.dz$$

$$+\frac{Eh^3}{24(1-\mu^2)}\iiint \left\{\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2} \quad \mu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2} \quad (1-\mu)\frac{\partial^2 w}{\partial xy}\right\}.\left\{\begin{matrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\,\frac{\partial^2 w}{\partial xy}\end{matrix}\right\} dx.dy.dz$$

$$U=\frac{1}{2}\frac{Eh}{(1-\mu^2)}\iiint \left\{\left(\frac{\partial u}{\partial x}+\mu\frac{\partial v}{\partial y}\right).\frac{\partial u}{\partial x}+\left(\mu\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right).\frac{\partial v}{\partial y}+\frac{(1-\mu)}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right\} dx.dy.dz$$

$$+\frac{Eh^3}{24(1-\mu^2)}\iiint \left\{\left(\frac{\partial^2 w}{\partial x^2}+\mu\frac{\partial^2 w}{\partial y^2}\right).\frac{\partial^2 w}{\partial x^2}+\left(\mu\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right).\ \frac{\partial^2 w}{\partial y^2}+(1-\mu)\frac{\partial^2 w}{\partial xy}.2\,\frac{\partial^2 w}{\partial xy}\right\} dx.dy.dz$$

3.2 Finite element method

For our analysis we will divide the plate into several slices or stripes along the plane of delamination/un-delamination. Each stripe will be considered as an element. So we will formulate the mid-plane displacements for each stipe.

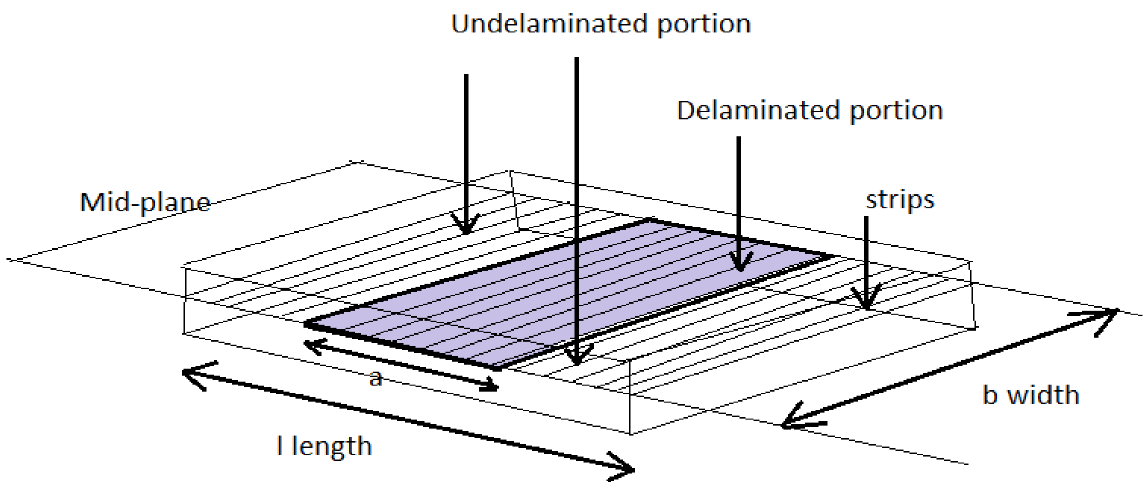


Diagram: Finite strip method

Now we will find the displacement functions for that strip (nodal lines)

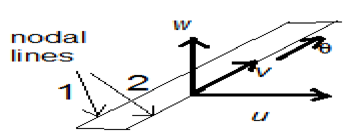


Diagram: nodal lines (strips)

Each nodal line can move in x, y and z direction so its mid-plane displacement functions can be found, depending upon its boundary conditions and degrees of freedom in each direction.

Displacement will be functions of x and y so, suppose

$$u(x,y)=\sum_n Fu(x)Y_{un}(y) \quad (8)$$

$$v(x,y)=\sum_n Fv(x)Y_{vn}(y) \quad (9)$$

$$w(x,y)=\sum_n Fw(x)Y_{wn}(y) \tag{10}$$

from the nodal strip we can see that nodal lines 1 and 2 can only move with single degree of freedom in x and y direction while in z direction there are 2 degrees of freedom because the nodal lines can also bend with respect to mid-plane.

degrees of freedom in different directions will be

$\{q\}u^T$ = $\{u1\ u2\}$ one degree of freedom for each nodal line

$\{q\}v^T$ = $\{v1\ v2\}$

$\{q\}w^T$ = $\{w1\ \ \theta1\ v2\ \ \theta2\}$

$\{S\}u$, $\{S\}v$ and $\{S\}w$ are the shape functions for u,v and w displacements. So displacements can be expressed in the form of shape function and degrees of freedoms.

$$u(x,y)=\{S\}u\{q\}u^T \tag{11}$$

$$v(x,y)=\{S\}v\{q\}v^T \tag{12}$$

$$w(x,y)=\{S\}w\{q\}w^T \tag{13}$$

if we will substitute these values in equations (1) and (2) and then putting those values in equation (7) , we will get the equation of strain energy

$$U=\frac{1}{2}\int\{q\}\mathrm{uv}^{\mathrm{T}}\{C\}\mathrm{uv}^{\mathrm{T}}[\mathrm{A}]\,\{q\}\mathrm{uv}\{C\}\mathrm{uv}\ d(vol.)$$

$$+\frac{1}{2}\int\{q\}\mathrm{w}^{\mathrm{T}}\{C\}\mathrm{w}^{\mathrm{T}}[\mathrm{D}]\,\{q\}\mathrm{w}\{C\}\mathrm{w}\ d(vol.)$$

Where$\{C\}$uvand $\{C\}$Ware the strain-displacement relation matrices which can be found by differentiating shape function matrices with respect to relevant variable x or y.

$\{q\}\mathrm{uv}$ =all in-plane degrees of freedom

$\{q\}\mathrm{w}$=degrees of freedom in z direction

Stiffness matrix can be found by performing partial differentiation of strain energy with respect to each degree of freedom$\{q\}\mathrm{i}$

$$[k_s]=\begin{bmatrix} k_{uv} & 0 \\ 0 & kuv \end{bmatrix}$$

$$\text{Where}\begin{bmatrix} k_{uv} \end{bmatrix}= \int \{C\}\mathrm{uv}^{\mathrm{T}}[\mathrm{A}]\{\,C\}\mathrm{uv}\,d(vol.),$$

$$\begin{bmatrix} k_w \end{bmatrix}= \int \{C\}\mathrm{w}^{\mathrm{T}}[\mathrm{A}]\{\,C\}\mathrm{w}\,d(vol.),$$

Now by putting the value of w from equation(13) to equation (6), we will get:

$$T=\frac{1}{2}\int \rho(\{\dot{q}\}\mathrm{w}^{\mathrm{T}}\{S\}\mathrm{w}^{\mathrm{T}}\{\dot{q}\}\mathrm{w}\{S\}\mathrm{w})^2\,d(vol.)$$

Now mass matrix can be found by performing the partial differentiation of T with respect to each degree of freedom$\{q\}\mathrm{i}$

$$[m_w]= \int \rho(\{S\}\mathrm{w}^{\mathrm{T}}\{S\}\mathrm{w})^2\,d(vol.)$$

By considering all the finite strips in the plate the equation of motion will become:

$$\begin{bmatrix} 0 & 0 \\ 0 & M_w \end{bmatrix} \begin{Bmatrix} \ddot{Q}_{uv} \\ \ddot{Q}_w \end{Bmatrix} + \begin{bmatrix} k_{uv} & k_{uvw} \\ k_{wuv} & k_w \end{bmatrix} \begin{Bmatrix} Q_{uv} \\ Q_w \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{14}$$

Where $[M_w]$ is global mass matrix ,

$[k_w]$ is the global bending stiffness matrix,

$[k_{uv}]$ is the global in-plane stiffness matrix

$[k_{uvw}]$ and $[k_{wuv}]$are the global in-plane/bending stiffness coupling matrix

$[Q_{uv}]$ in-plane degrees of freedom u_i and v_i

$[Q_{uv}]$out of plane degrees of freedomw_i and θ_i

By eliminating the in-plane degrees of freedom we will get:

$$[M_w]\{\ddot{Q}_w\} + ([k_w] - [k_{wuv}][k_{uv}]^{-1}[k_{uvw}])\{Q_w\} = \{0\} \tag{15}$$

If the motion of plate is represented by an exponential function of time:

$$\{Q_w\} = \{\overline{Q_w}\}e^{i\omega t}$$

Putting this value in equation (15)

$$[k_w] - [k_{wuv}][k_{uv}]^{-1}[k_{uvw}] - \omega^2[M_w] = \{0\}$$

$\{\overline{Q_w}\}$ contains all the out of plane degrees of freedom [7].

CHAPTER ~4

4.Vibration analysis using Ansys13.0

4.1Steps followed for analysis:

We have already derived the equations of the motion in the formulation section, now we will do the vibration analysis of the delaminated plates using Ansys13.0 and compare the results with results obtained by equations of motion .We will use these steps for the analysis of the delaminated plates:

1. We have designed the plates using CatiaV5 for each analysis. Delamination variables are varied according to requirements of analysis. Dimensions are same for all the plates but delamination variables such as length of delamination, position of delamination are varied for each analysis setup.

Thin Plate (lamina) specifications:

1. Dimensions of the plate:
   Length 50 mm
   Width 30mm
   Height 4 mm

2. Plate material:
   Structural steel

Properties	
Volume	5932.5 mm³
Mass	4.657e-002 kg
Centroid X	2.0111 mm
Centroid Y	15. mm
Centroid Z	-25. mm
Moment of Inertia Ip1	13.216 kg·mm²
Moment of Inertia Ip2	9.7854 kg·mm²
Moment of Inertia Ip3	3.555 kg·mm²

3. Delamination variables:

- Length of delamination (a)
- Position of delamination (t)
  Distance of the delaminated plane from the upper plane of the plate

2. Then these plate files saved in STEP (.stp) format are exported to Ansys13.0 Workbench Modal analysis. Where we have given the boundary conditions and meshed the geometry. After that each plate design is solved for vibration up to 6 modes.

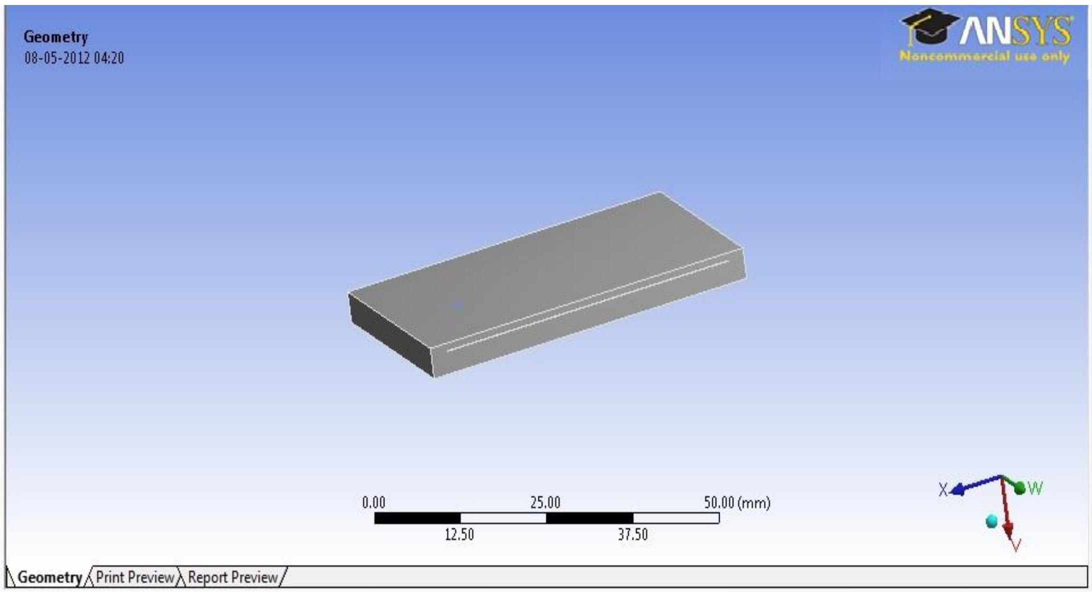


Diagram: Plate with through width delamination (a/l)=0.9 and (t/h)=0.25

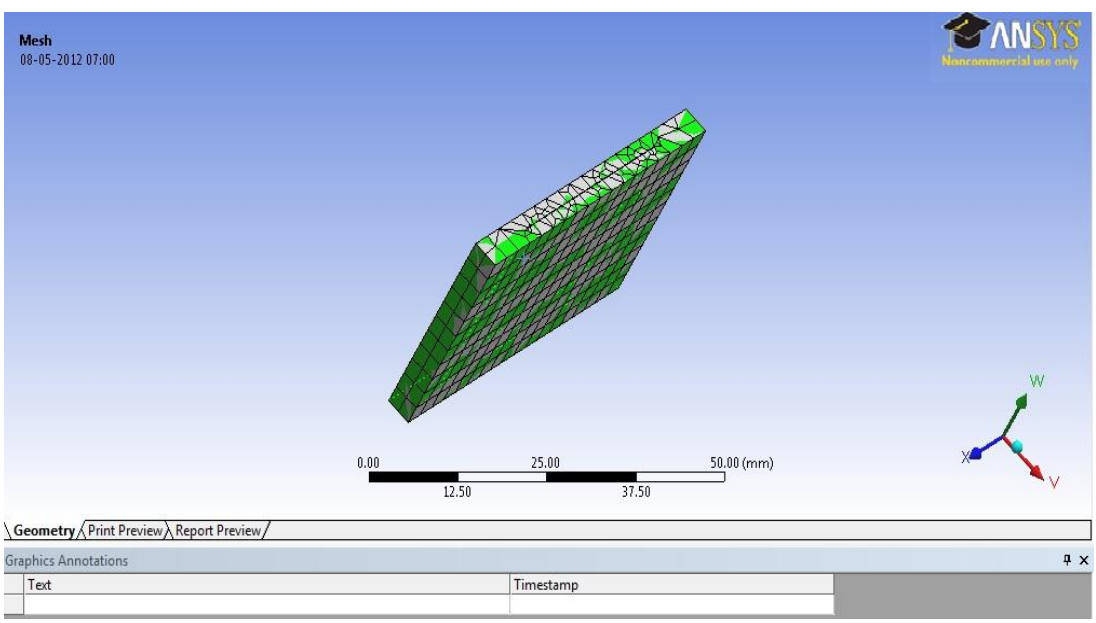
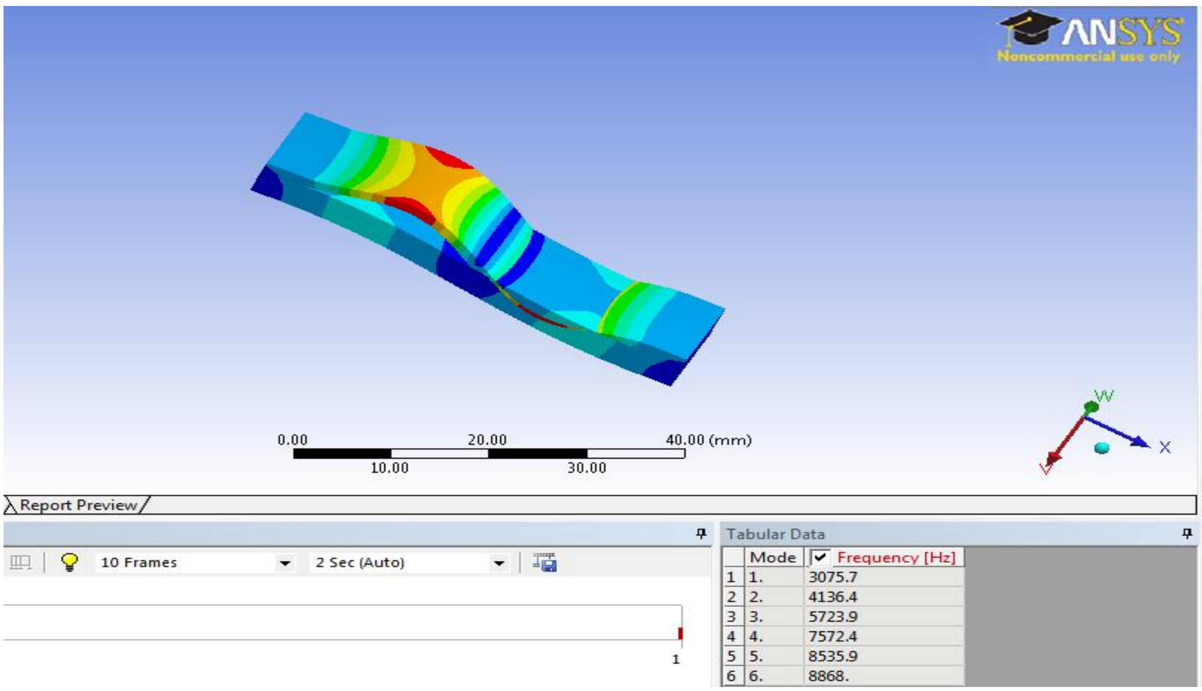


Diagram: Mesh of a delaminated plate{ (a/l)=0.5 and (t/h)=0.5}

3. Frequencies are found for each mode and tabulated for comparison and discussion.

4. We can see the simulations for deformation patterns for each plate and understand the effects of delamination on its natural frequency.



	Mode	Frequency [Hz]
1	1.	3075.7
2	2.	4136.4
3	3.	5723.9
4	4.	7572.4
5	5.	8535.9
6	6.	8868.

Diagram: Deformation patterns for delaminated plated { (a/l)=0.8 and (t/h)=0.25 4th mode} Frequencies are also given (up to 6th modes of vibration).

5. Frequency related data is collected to compare and discuss the results.

4.2 Data obtained from analysis

Analysis has been done on 19 plates of same dimensions but having different delamination variables i.e. (a/l) and (t/h). From this data we will try to correlate the frequency with delamination variables. Data obtained from analysis are as follows:

Sl.no.	plate type	Delamination variables			Frequency	*f0*	*f/fo*
	plate 1	(a/l)	(t/h)	mode	Hz		
1	without delamination	0	..	1	5560	5560	1
2	without delamination	0	..	2	9294.3	9294.3	1
3	without delamination	0	..	3	13236	13236	1
4	without delamination	0	..	4	20698	20698	1
5	without delamination	0	..	5	21487	21487	1
6	without delamination	0	..	6	27571	27571	1
	plate 2						
7	with delamination	0.1	0.25	1	5553	5560	0.998741
8	with delamination	0.1	0.25	2	9293.4	9294.3	0.999903
9	with delamination	0.1	0.25	3	13161	13236	0.994334
10	with delamination	0.1	0.25	4	20693	20698	0.999758
11	with delamination	0.1	0.25	5	21309	21487	0.991716
12	with delamination	0.1	0.25	6	27381	27571	0.993109
	plate 3						
13	with delamination	0.2	0.25	1	5547.6	5560	0.99777
14	with delamination	0.2	0.25	2	9243.9	9294.3	0.994577
15	with delamination	0.2	0.25	3	12757	13236	0.963811
16	with delamination	0.2	0.25	4	20528	20698	0.991787
17	with delamination	0.2	0.25	5	20697	21487	0.963234
18	with delamination	0.2	0.25	6	26216	27571	0.950854

	plate 4						
19	with delamination	0.3	0.25	1	5519.6	5560	0.992734
20	with delamination	0.3	0.25	2	9099.5	9294.3	0.979041
21	with delamination	0.3	0.25	3	12064	13236	0.911454
22	with delamination	0.3	0.25	4	19415	20698	0.938013
23	with delamination	0.3	0.25	5	20512	21487	0.954624
24	with delamination	0.3	0.25	6	21160	27571	0.767473
	plate 5						
25	with delamination	0.4	0.25	1	5455.5	5560	0.981205
26	with delamination	0.4	0.25	2	8672.2	9294.3	0.933067
27	with delamination	0.4	0.25	3	11070	13236	0.836355
28	with delamination	0.4	0.25	4	13054	20698	0.630689
29	with delamination	0.4	0.25	5	14562	21487	0.677712
30	with delamination	0.4	0.25	6	16798	27571	0.609263
	plate 6						
31	with delamination	0.5	0.25	1	5234.5	5560	0.941457
32	with delamination	0.5	0.25	2	7638.5	9294.3	0.821848
33	with delamination	0.5	0.25	3	8886.9	13236	0.671419
34	with delamination	0.5	0.25	4	10233	20698	0.494396
35	with delamination	0.5	0.25	5	11062	21487	0.514823
36	with delamination	0.5	0.25	6	13127	27571	0.476116
	plate 7						
37	with delamination	0.6	0.25	1	4679.7	5560	0.841673
38	with delamination	0.6	0.25	2	6197	9294.3	0.666753
39	with delamination	0.6	0.25	3	6957.8	13236	0.525672
40	with delamination	0.6	0.25	4	9391.6	20698	0.453744
41	with delamination	0.6	0.25	5	9779.2	21487	0.455122
42	with delamination	0.6	0.25	6	11026	27571	0.399913
	plate 8						
43	with delamination	0.7	0.25	1	3826.9	5560	0.688291
44	with delamination	0.7	0.25	2	4991.1	9294.3	0.537007
45	with delamination	0.7	0.25	3	6154.9	13236	0.465012
46	with delamination	0.7	0.25	4	8535.3	20698	0.412373
47	with delamination	0.7	0.25	5	9071.6	21487	0.42219
48	with delamination	0.7	0.25	6	9710.6	27571	0.3522

	plate 9						
49	with delamination	0.8	0.25	1	3075.7	5560	0.553183
50	with delamination	0.8	0.25	2	4136.4	9294.3	0.445047
51	with delamination	0.8	0.25	3	5723.9	13236	0.432449
52	with delamination	0.8	0.25	4	7572.4	20698	0.365852
53	with delamination	0.8	0.25	5	8535.9	21487	0.397259
54	with delamination	0.8	0.25	6	8868	27571	0.321642

	plate 10						
55	with delamination	0.9	0.25	1	2479.6	5560	0.445971
56	with delamination	0.9	0.25	2	3492.7	9294.3	0.375789
57	with delamination	0.9	0.25	3	5294.3	13236	0.399992
58	with delamination	0.9	0.25	4	6460.1	20698	0.312112
59	with delamination	0.9	0.25	5	8025.2	21487	0.373491
60	with delamination	0.9	0.25	6	8270.2	27571	0.29996

	plate 11						
61	with delamination	0.1	0.5	1	5559.4	5560	0.999892
62	with delamination	0.1	0.5	2	9291.4	9294.3	0.999688
63	with delamination	0.1	0.5	3	13086	13236	0.988667
64	with delamination	0.1	0.5	4	20690	20698	0.999613
65	with delamination	0.1	0.5	5	21152	21487	0.984409
66	with delamination	0.1	0.5	6	27285	27571	0.989627

	plate 12						
67	with delamination	0.2	0.5	1	5721.4	5560	1.029029
68	with delamination	0.2	0.5	2	9405.6	9294.3	1.011975
69	with delamination	0.2	0.5	3	12659	13236	0.956407
70	with delamination	0.2	0.5	4	20396	20698	0.985409
71	with delamination	0.2	0.5	5	21560	21487	1.003397
72	with delamination	0.2	0.5	6	26462	27571	0.959777

	plate 13						
73	with delamination	0.3	0.5	1	5553.7	5560	0.998867
74	with delamination	0.3	0.5	2	9082.9	9294.3	0.977255
75	with delamination	0.3	0.5	3	11109	13236	0.839302
76	with delamination	0.3	0.5	4	18001	20698	0.869698
77	with delamination	0.3	0.5	5	20669	21487	0.96193
78	with delamination	0.3	0.5	6	23655	27571	0.857967
	plate 14						
79	with delamination	0.4	0.5	1	5507.6	5560	0.990576
80	with delamination	0.4	0.5	2	8769.9	9294.3	0.943578
81	with delamination	0.4	0.5	3	9753.5	13236	0.736892
82	with delamination	0.4	0.5	4	16497	20698	0.797034
83	with delamination	0.4	0.5	5	20430	21487	0.950807
84	with delamination	0.4	0.5	6	20588	27571	0.746727
	plate 15						
85	with delamination	0.5	0.5	1	5404.3	5560	0.971996
86	with delamination	0.5	0.5	2	8300.9	9294.3	0.893117
87	with delamination	0.5	0.5	3	8637.4	13236	0.652569
88	with delamination	0.5	0.5	4	13937	20698	0.67335
89	with delamination	0.5	0.5	5	15596	21487	0.725834
90	with delamination	0.5	0.5	6	16016	27571	0.5809
	plate 16						
91	with delamination	0.6	0.5	1	5169.4	5560	0.929748
92	with delamination	0.6	0.5	2	7680.9	9294.3	0.82641
93	with delamination	0.6	0.5	3	7825.1	13236	0.591198
94	with delamination	0.6	0.5	4	9911.9	20698	0.478882
95	with delamination	0.6	0.5	5	11974	21487	0.557267
96	with delamination	0.6	0.5	6	15043	27571	0.54561

	plate 17						
97	with delamination	0.7	0.5	1	4867.3	5560	0.875414
98	with delamination	0.7	0.5	2	7057.4	9294.3	0.759326
99	with delamination	0.7	0.5	3	7305.4	13236	0.551934
100	with delamination	0.7	0.5	4	7599.8	20698	0.367176
101	with delamination	0.7	0.5	5	9623.2	21487	0.447861
102	with delamination	0.7	0.5	6	14693	27571	0.532915
	plate 18						
103	with delamination	0.8	0.5	1	4461.2	5560	0.802374
104	with delamination	0.8	0.5	2	5880.4	9294.3	0.632689
105	with delamination	0.8	0.5	3	6431.3	13236	0.485895
106	with delamination	0.8	0.5	4	6975	20698	0.336989
107	with delamination	0.8	0.5	5	7855.4	21487	0.365588
108	with delamination	0.8	0.5	6	14256	27571	0.517065
	plate 19						
109	with delamination	0.9	0.5	1	4035.8	5560	0.725863
110	with delamination	0.9	0.5	2	4670.7	9294.3	0.502534
111	with delamination	0.9	0.5	3	5903.8	13236	0.446041
112	with delamination	0.9	0.5	4	6595.3	20698	0.318644
113	with delamination	0.9	0.5	5	6816.3	21487	0.317229
114	with delamination	0.9	0.5	6	12772	27571	0.46324

A bigger plate with dimensions; 200mmx100mmx20mm of same material was also considered and analysed for some cases of delamination variables to understand that how the size affects the impact of delamination variables on frequency.

		Delamination variable			
sl.no		(a/l)	(t/h)	mode	frequency
	plate 20				
1	Un-delaminated	0	..	1	1692.1
2	Un-delaminated	0	..	2	3071.2
3	Un-delaminated	0	..	3	3810.9
4	Un-delaminated	0	..	4	4759
5	Un-delaminated	0	..	5	6727.4
6	Un-delaminated	0	..	6	8148.7
	plate 21				
7	delaminated	0.5	0.25	1	1648.6
8	delaminated	0.5	0.25	2	2589.5
9	delaminated	0.5	0.25	3	2836.8
10	delaminated	0.5	0.25	4	3124.8
11	delaminated	0.5	0.25	5	3679.7
12	delaminated	0.5	0.25	6	
	plate 22				
13	delaminated	0.7	0.25	1	1229.9
14	delaminated	0.7	0.25	2	1722.8
15	delaminated	0.7	0.25	3	1937.3
16	delaminated	0.7	0.25	4	2645.1
17	delaminated	0.7	0.25	5	3074.9
18	delaminated	0.7	0.25	6	3896.3
	plate 23				
19	delaminated	0.5	0.5	1	1632.7
20	delaminated	0.5	0.5	2	2594.4
21	delaminated	0.5	0.5	3	2742.3
22	delaminated	0.5	0.5	4	4132.6
23	delaminated	0.5	0.5	5	4630.9
24	delaminated	0.5	0.5	6	5008.1

CHAPTER~5

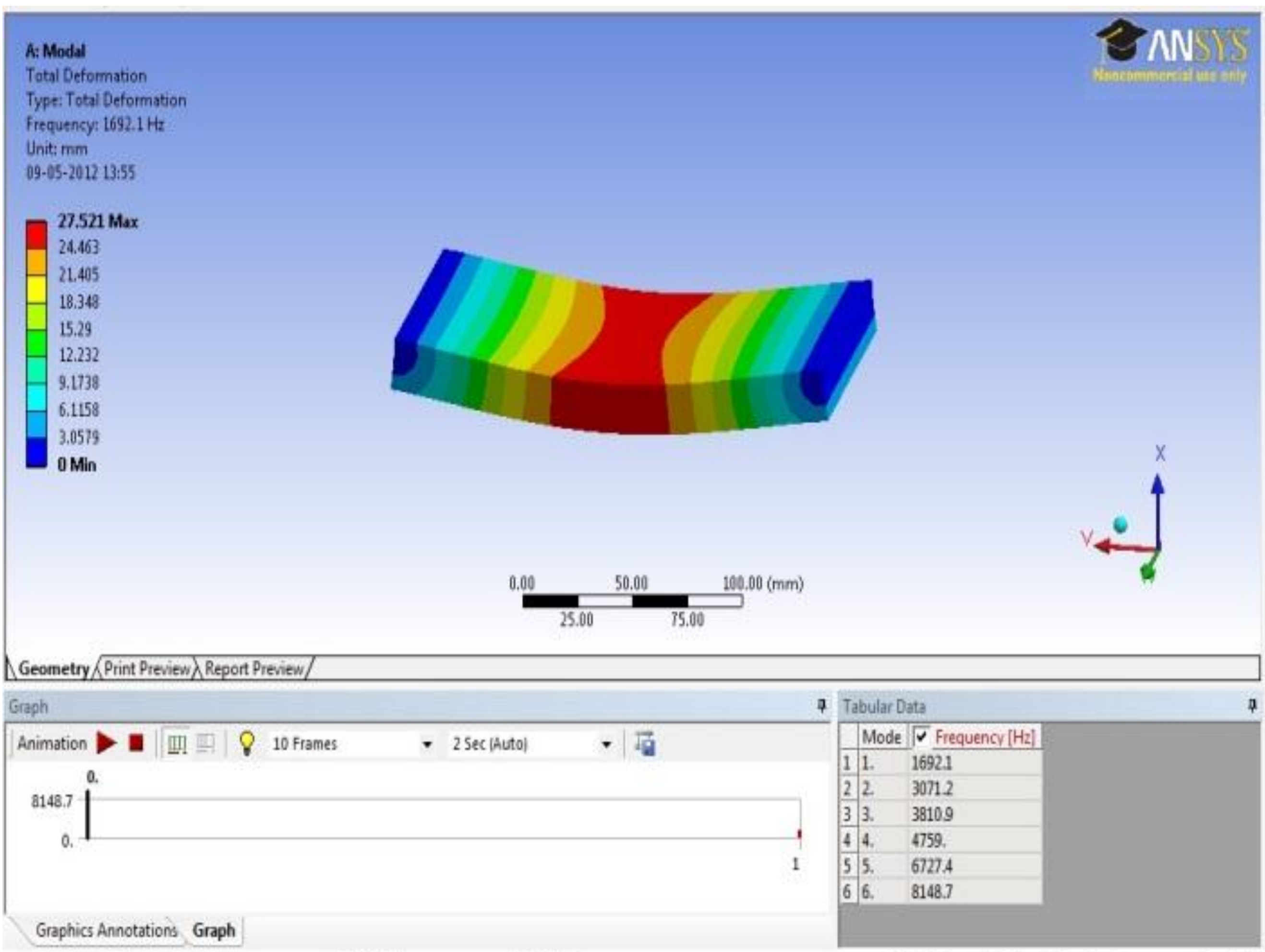
5. Results and Discussion

1. Deformation patterns for different cases of delamination:

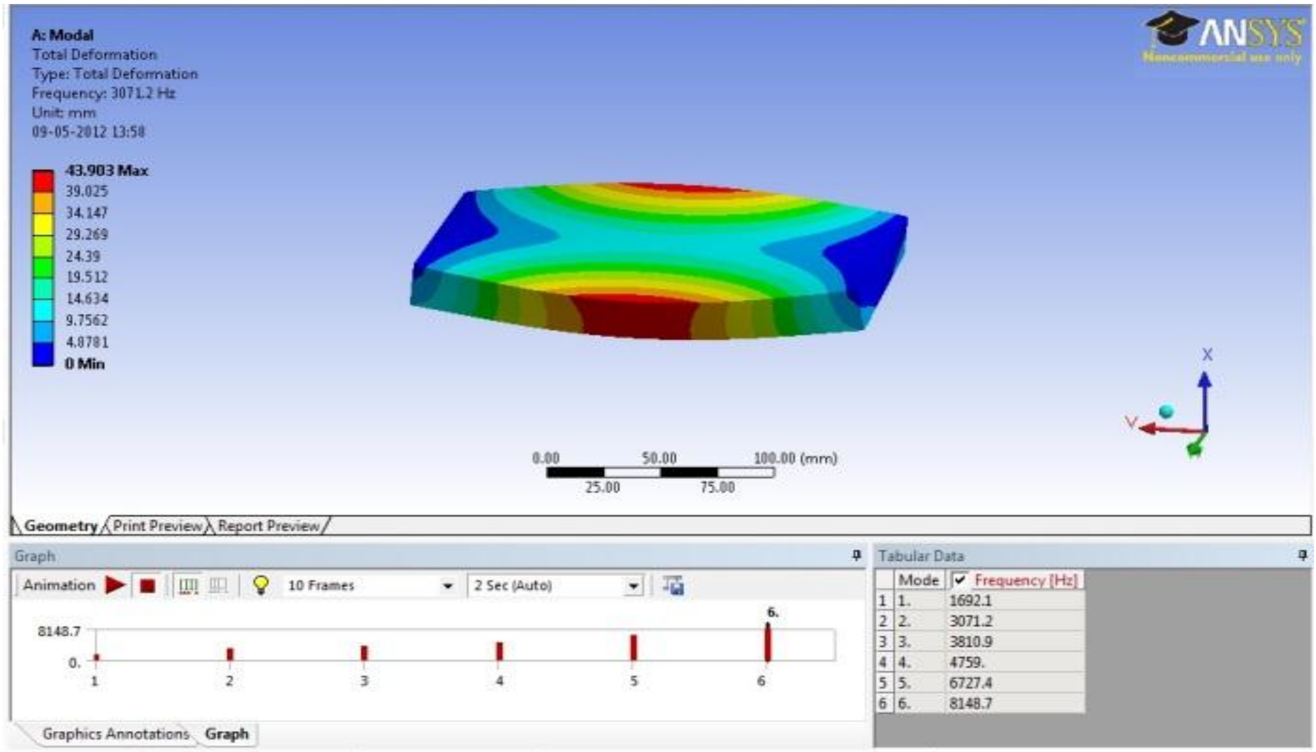
Some deformation patterns from Ansys13.0 analysis are presented to have a better understanding of delamination.

1st case: plate 20, Dimensions: 200mm x 100mm x 20mm, (a/l) = 0, Un-delaminated

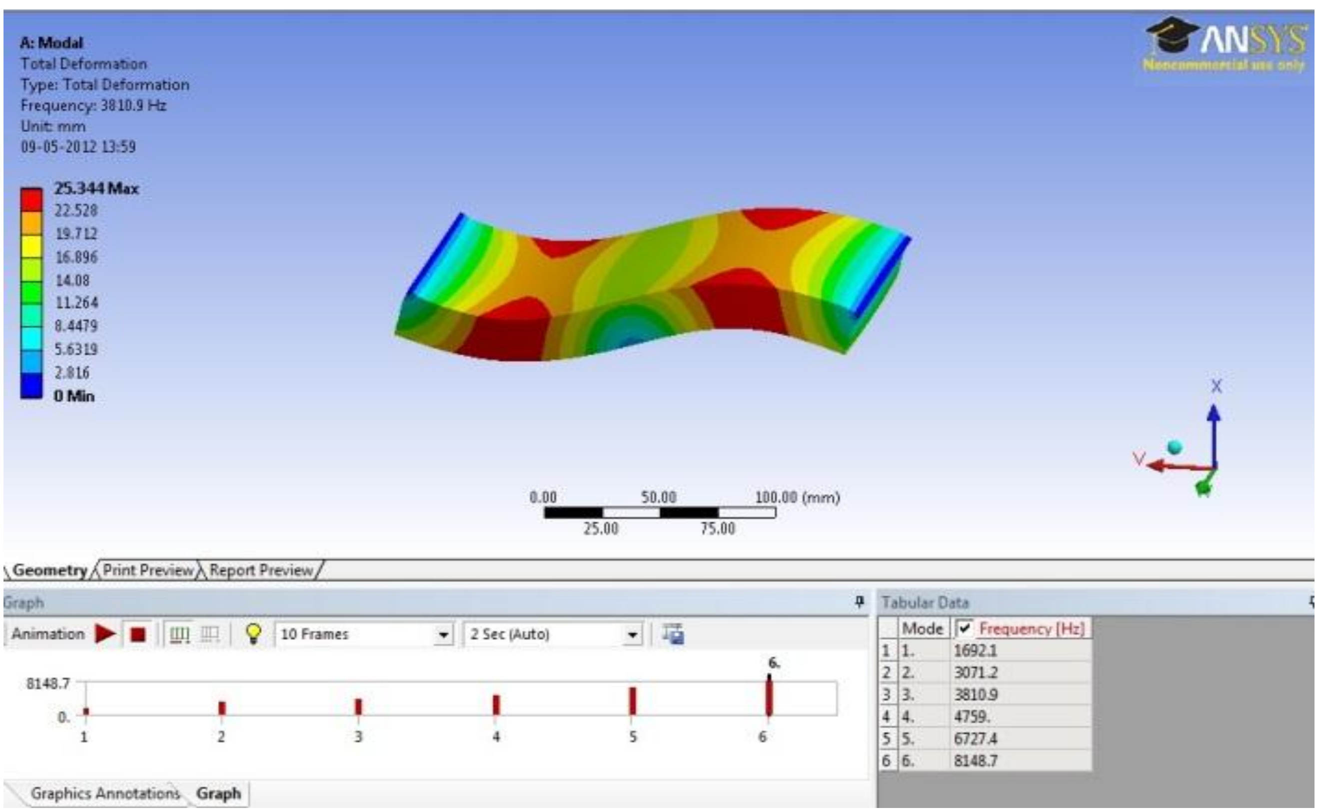
1st mode of vibration



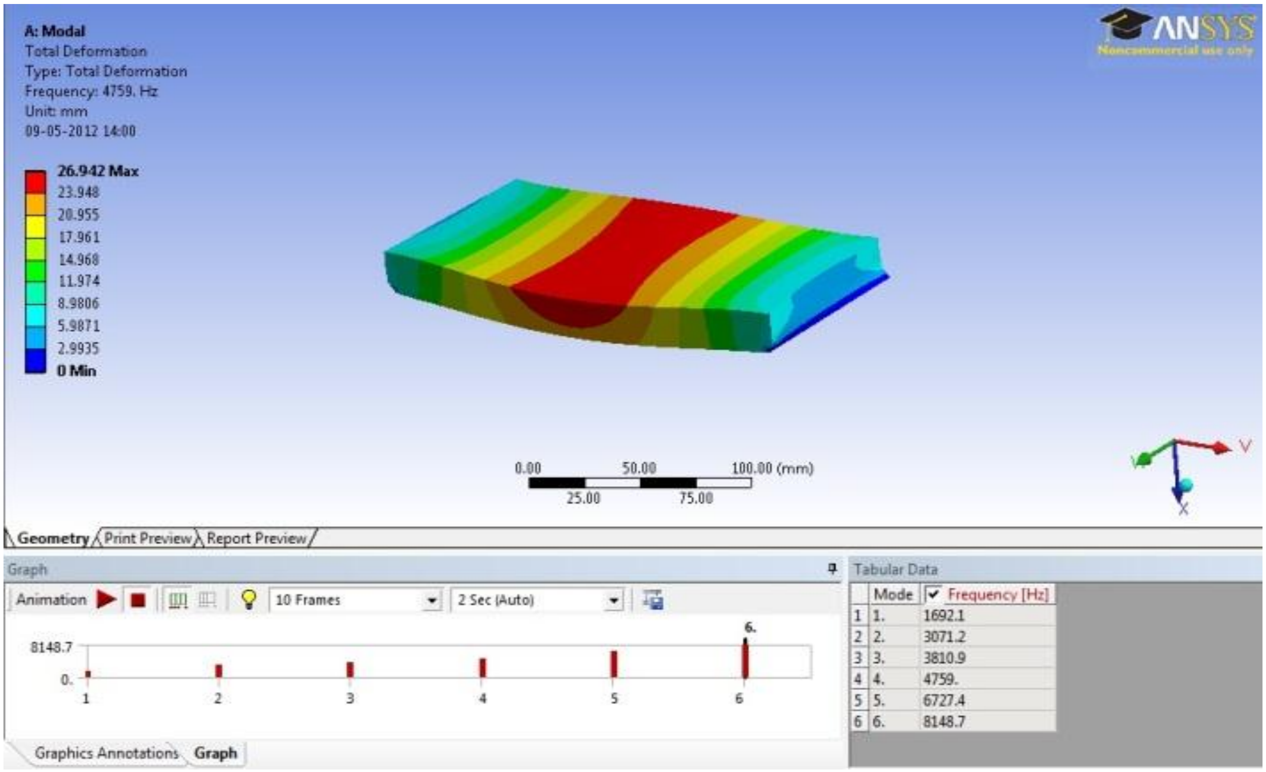
2nd mode of vibration



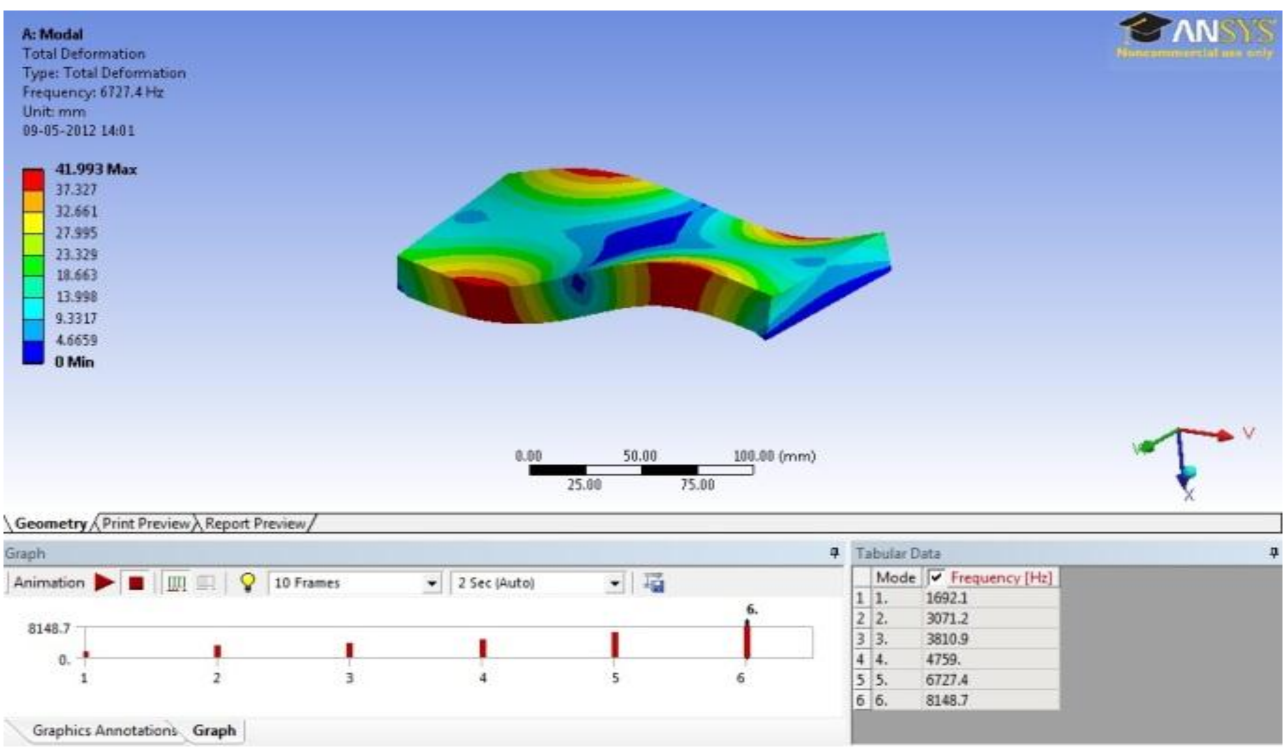
3rd mode of vibration



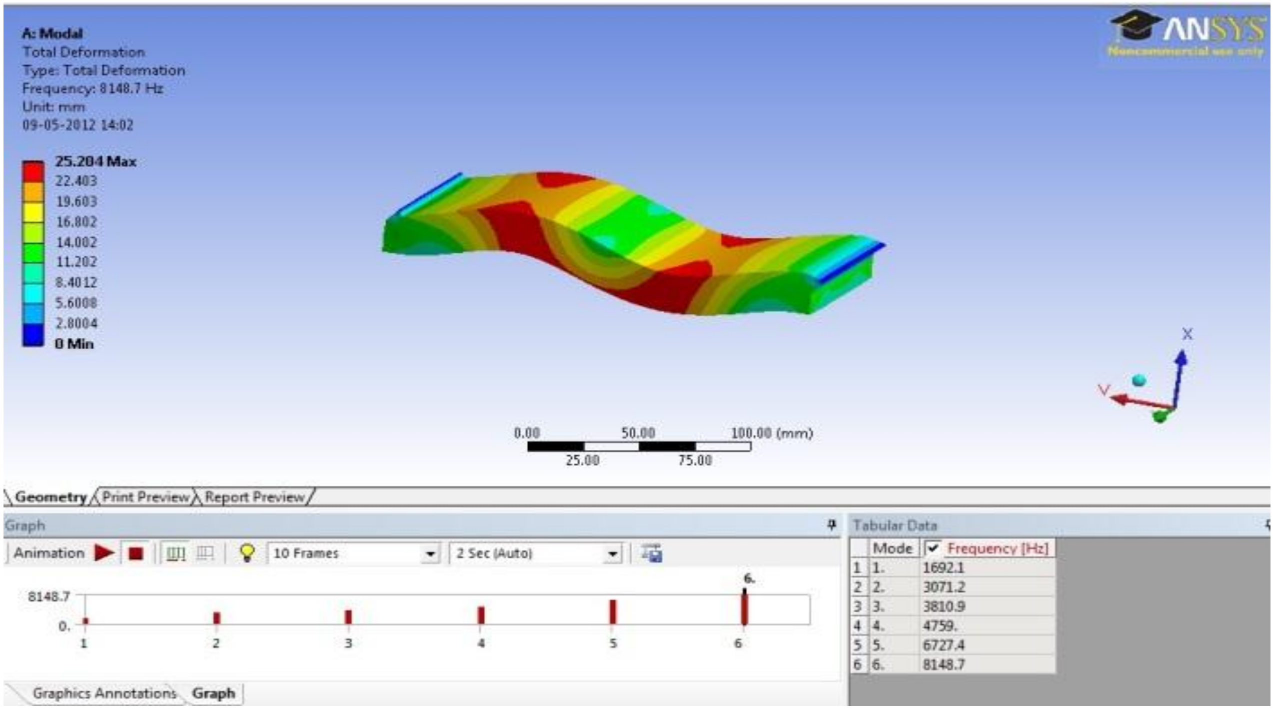
4^{th} mode of vibration



5^{th} mode of vibration

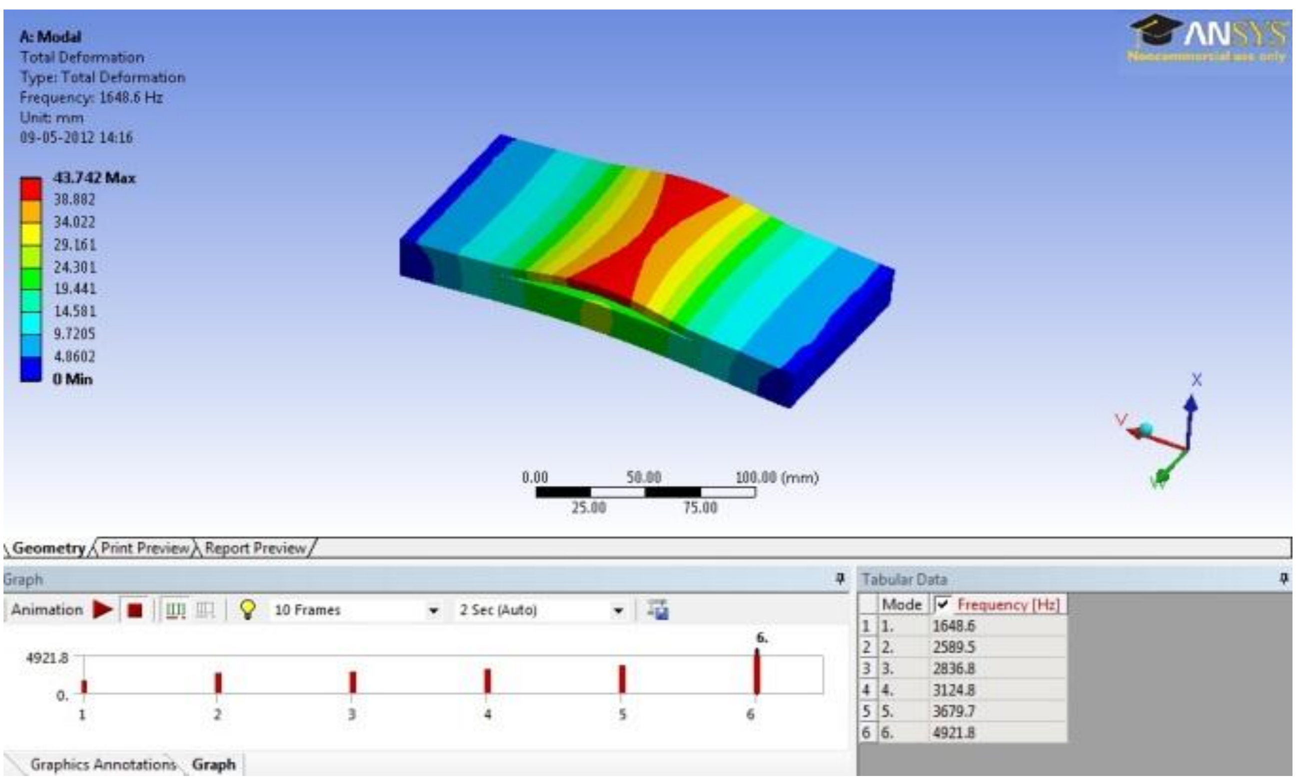


6th mode of vibration

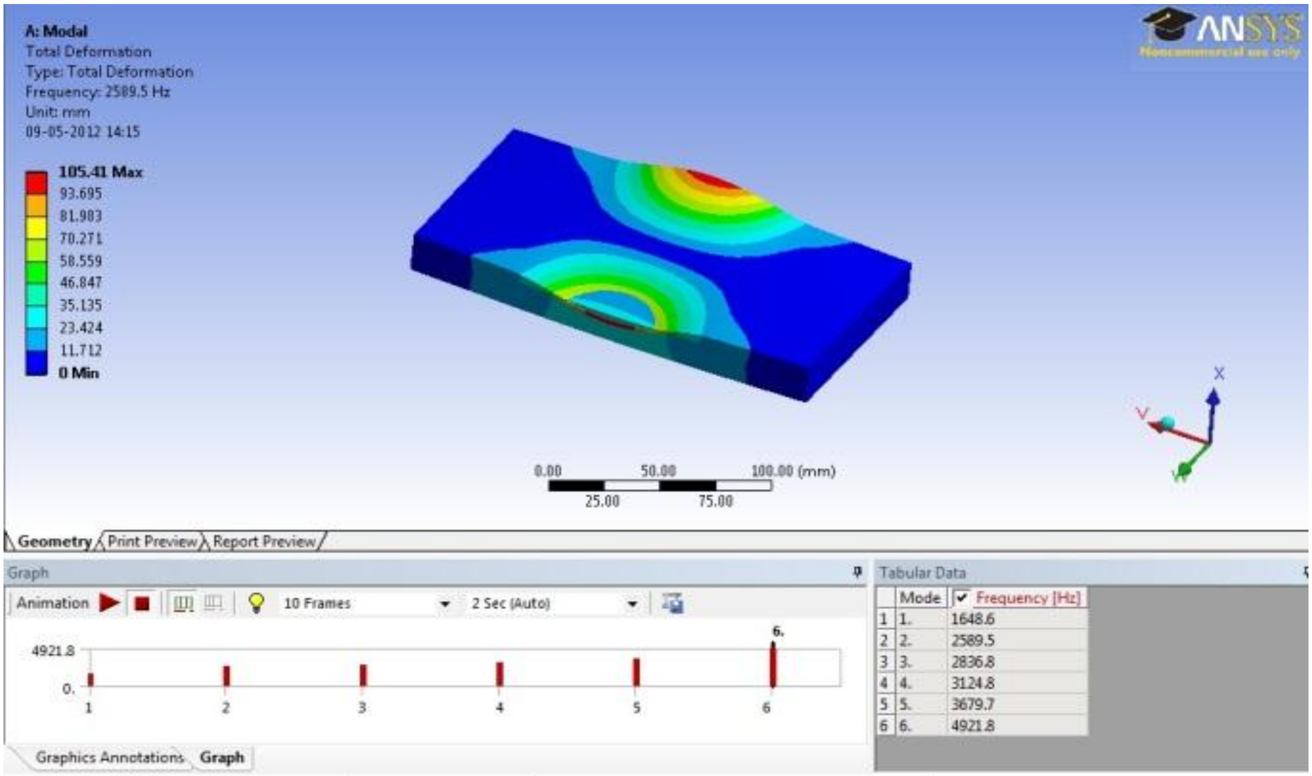


2nd case: plate 21, Dimensions: 200mm x 100mm x 20mm, (a/l) = 0.5, (t/h)=0.25

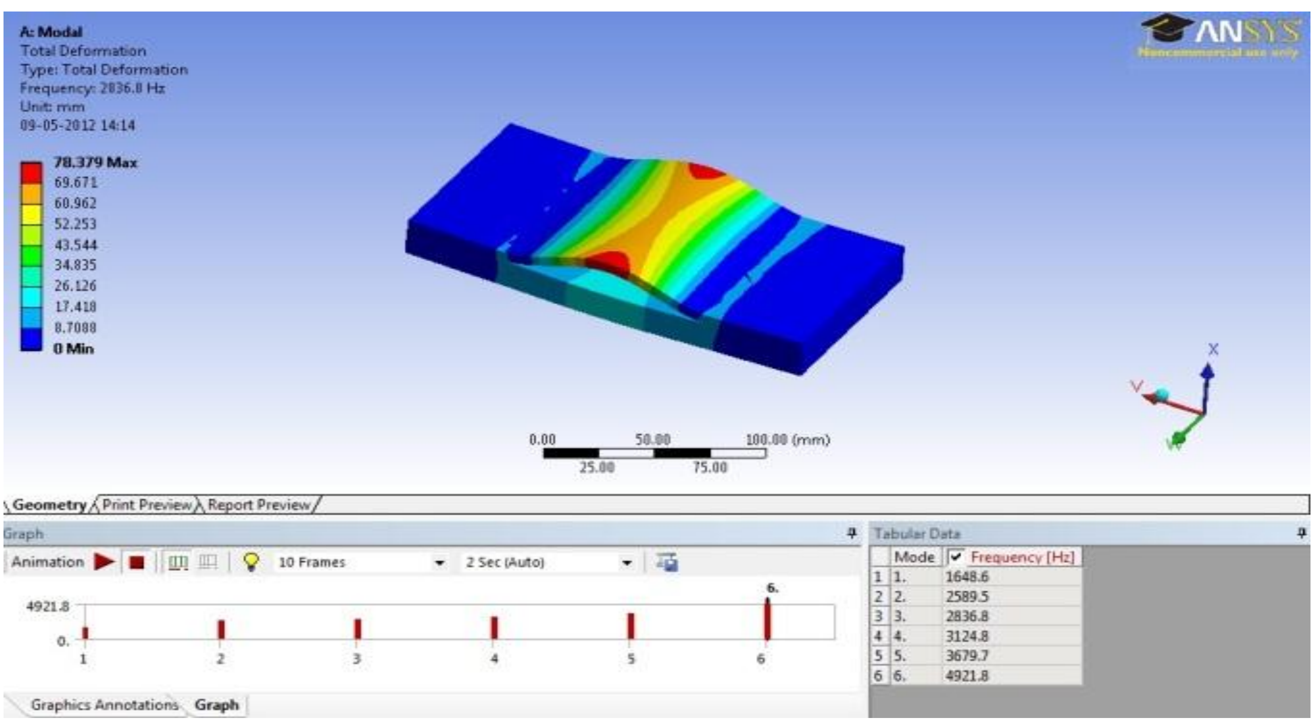
1st mode of vibration



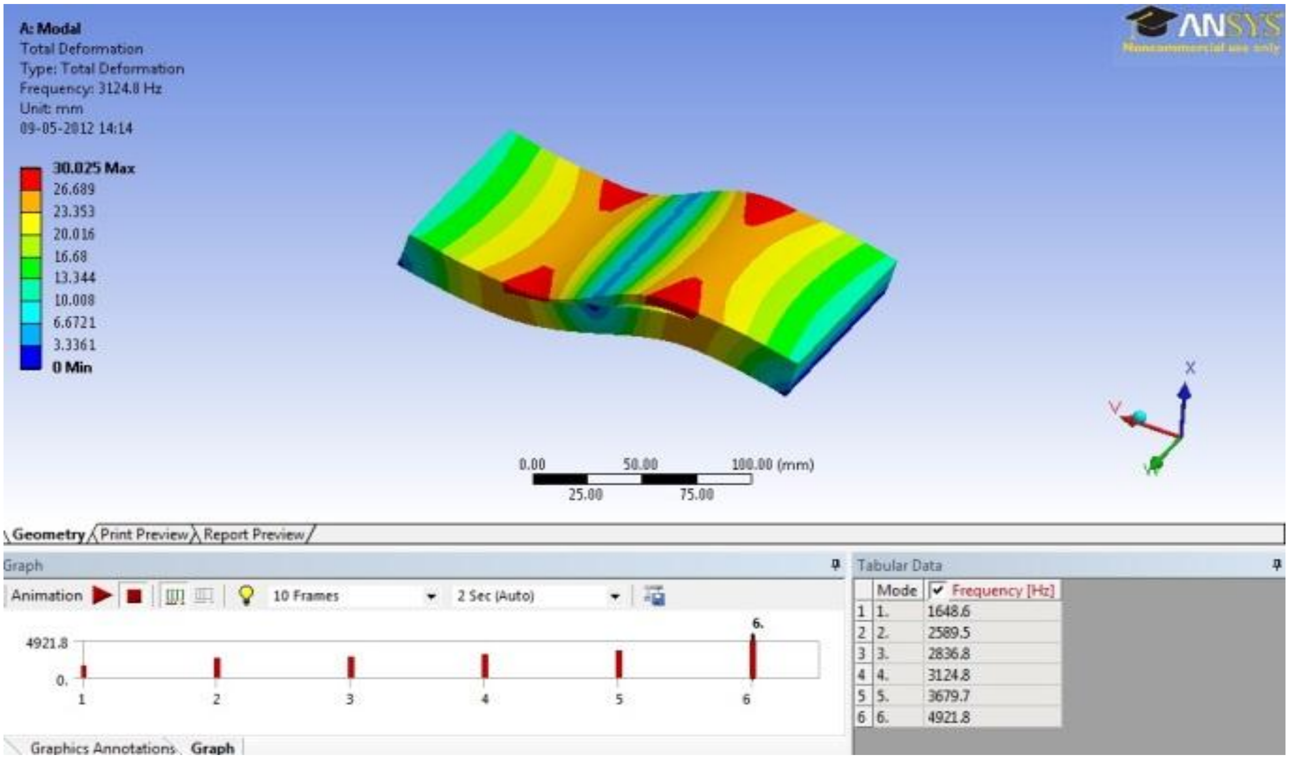
2nd mode of vibration



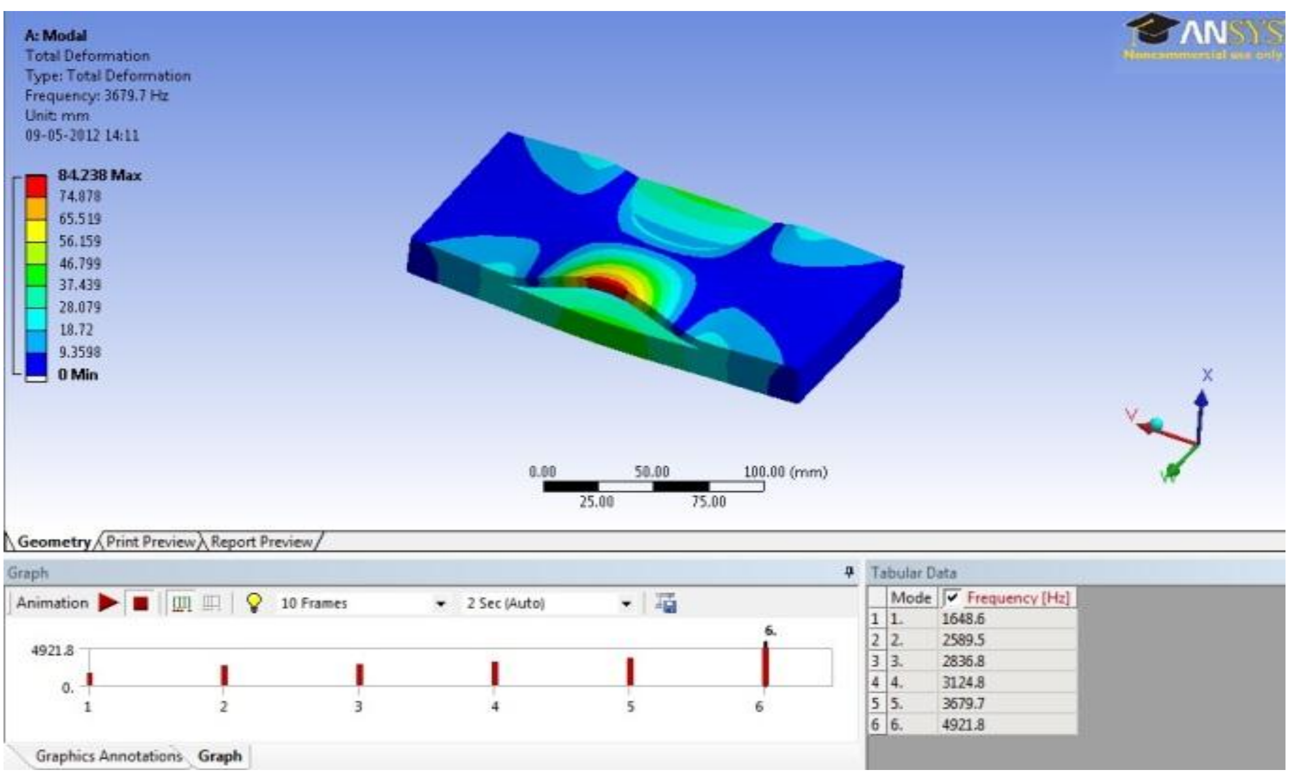
3rd mode of vibration



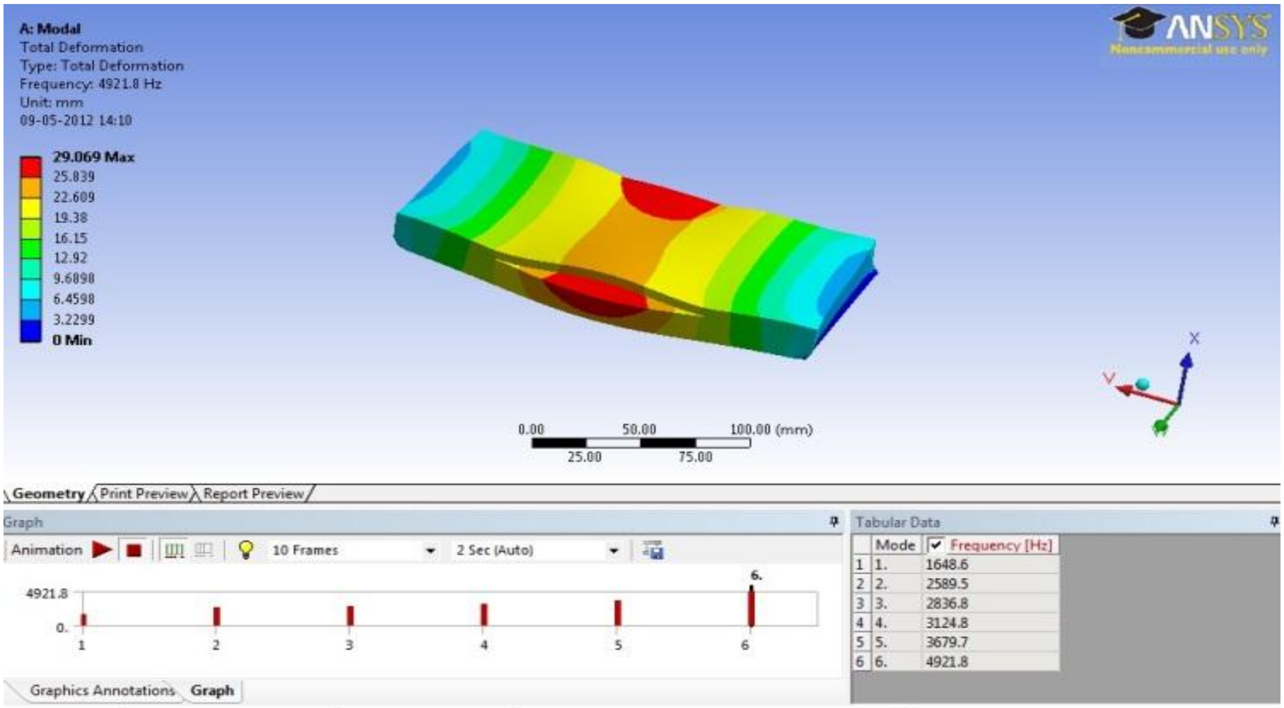
4^{th} mode of vibration



5^{th} mode of vibration

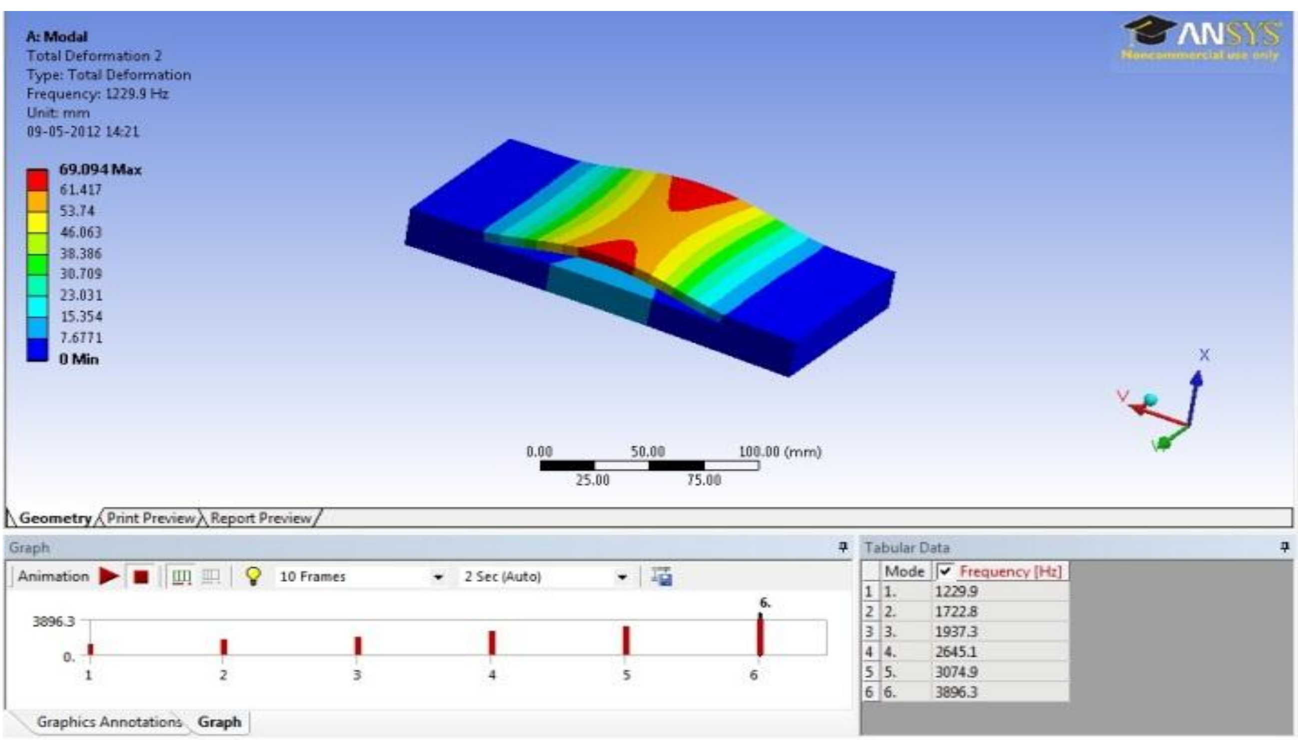


6th mode of vibration

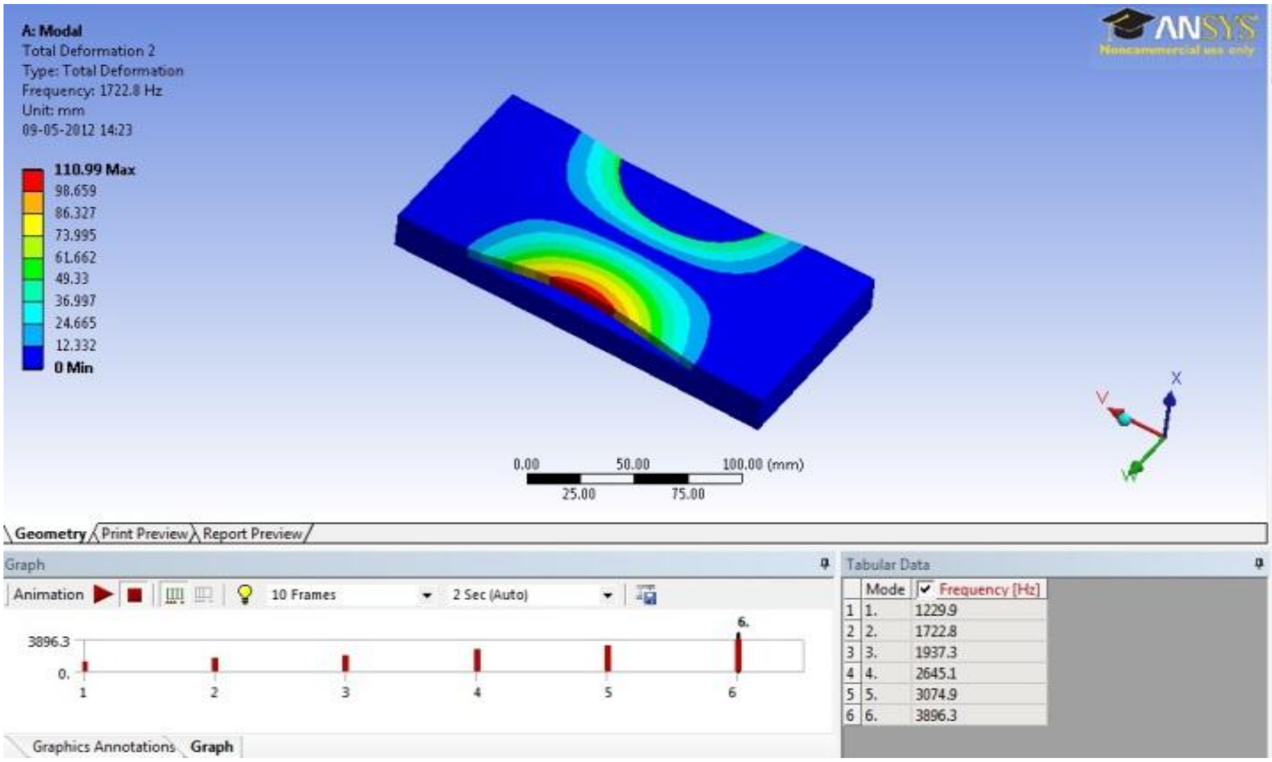


3rd case: plate 22, Dimensions: 200mm x 100mm x 20mm, (a/l) =0.7, (t/h) = 0.25

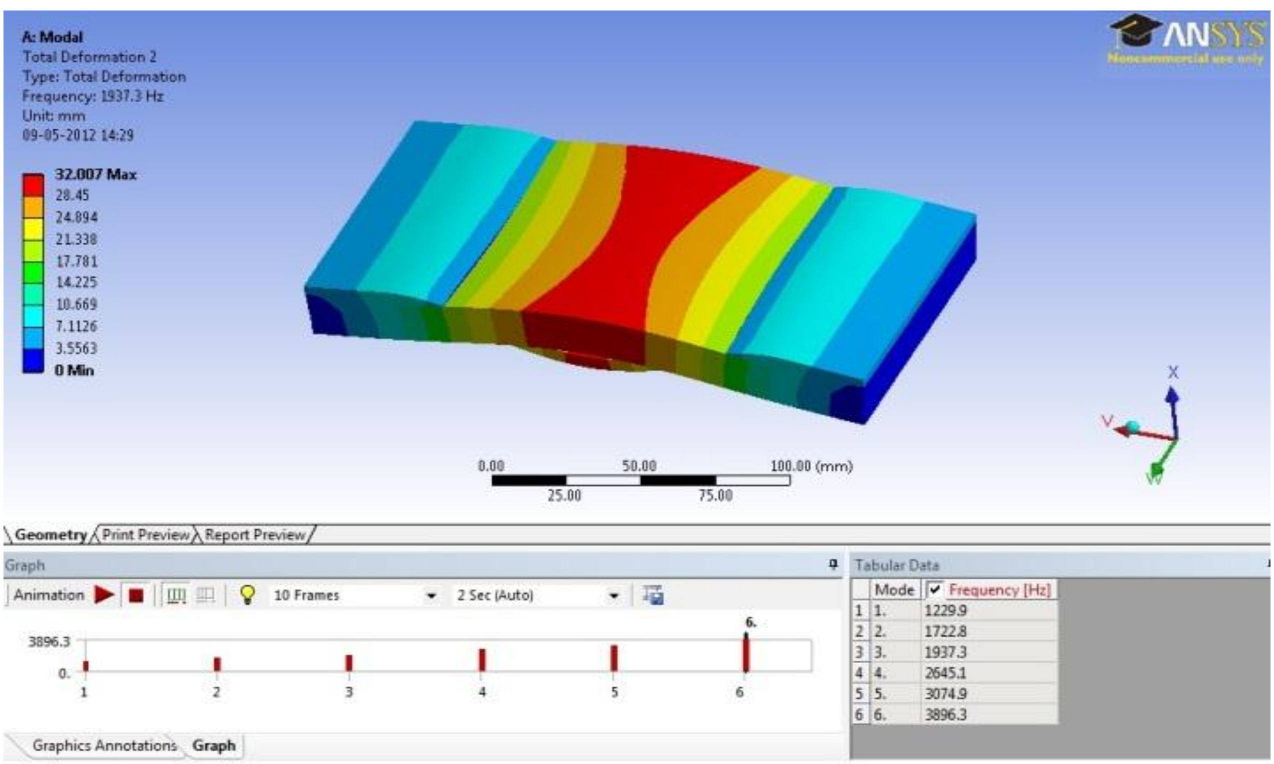
1st mode of vibration



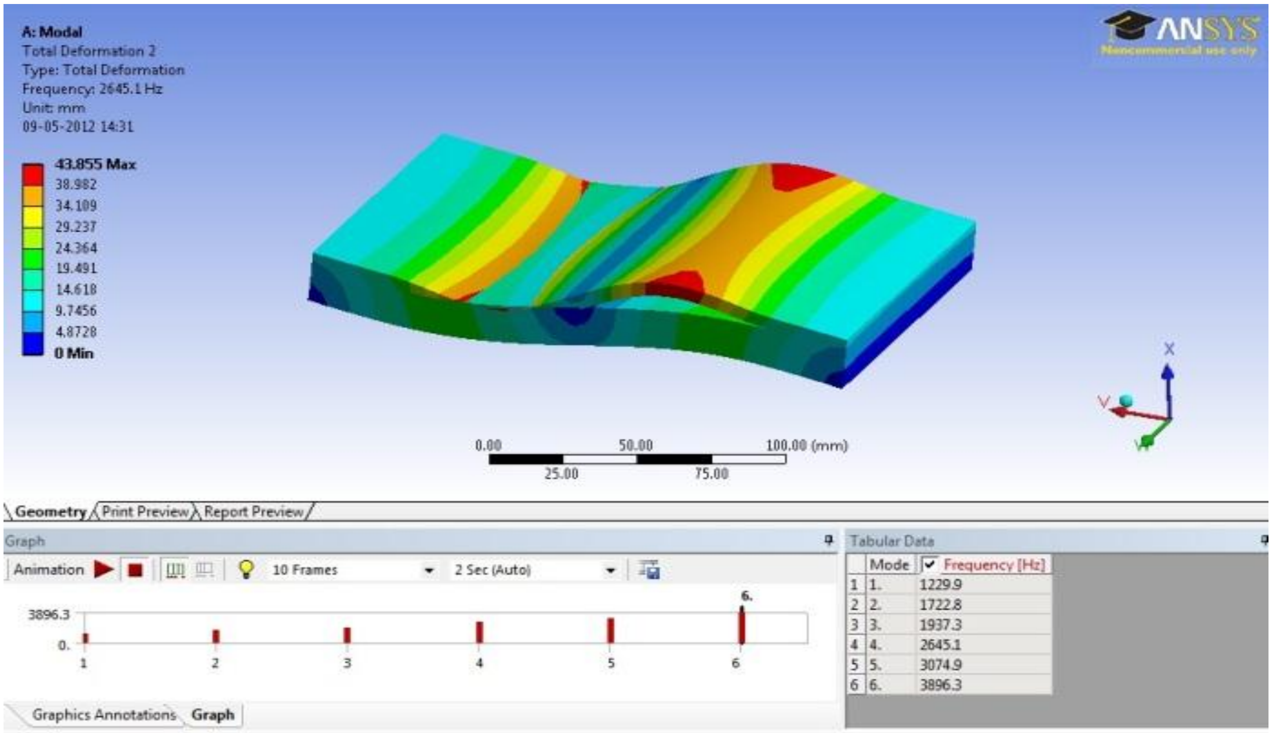
2nd mode of vibration



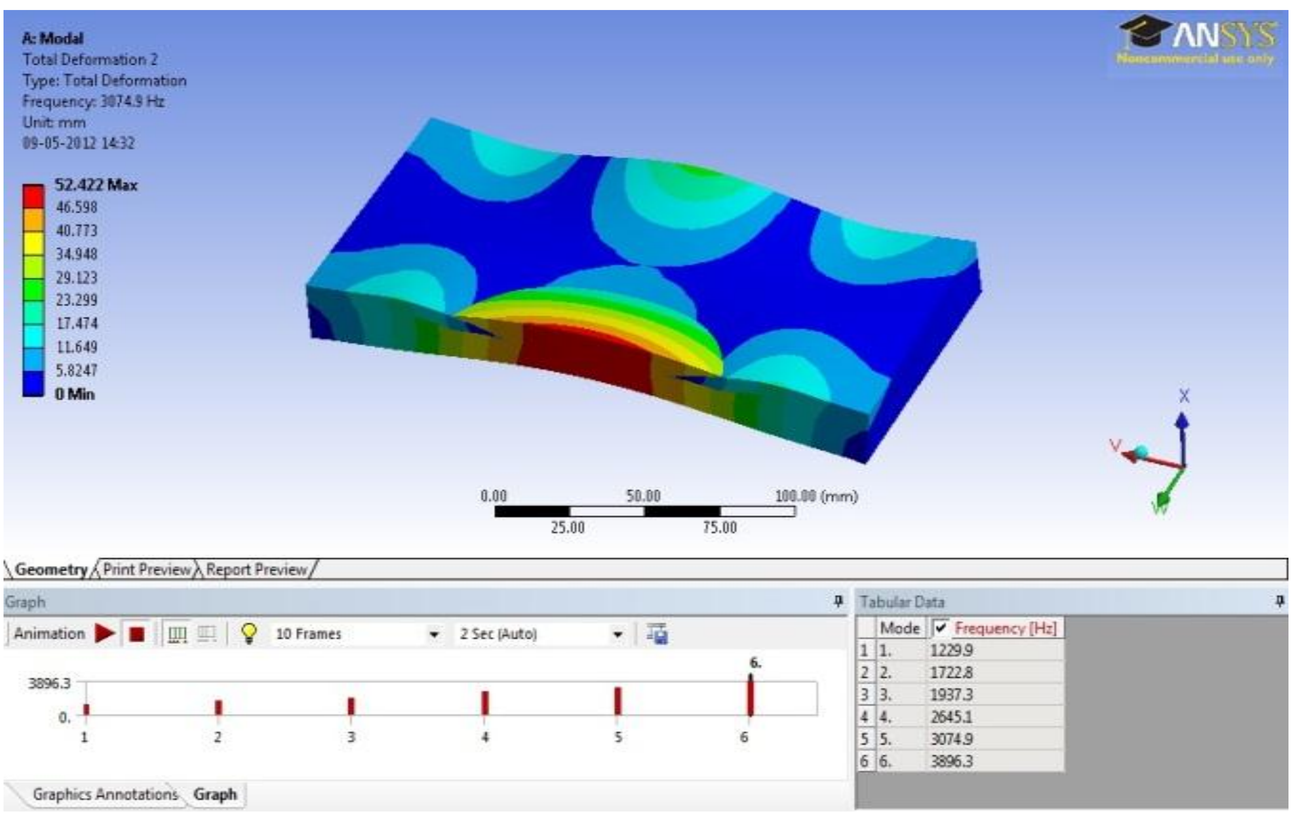
3rd mode of vibration



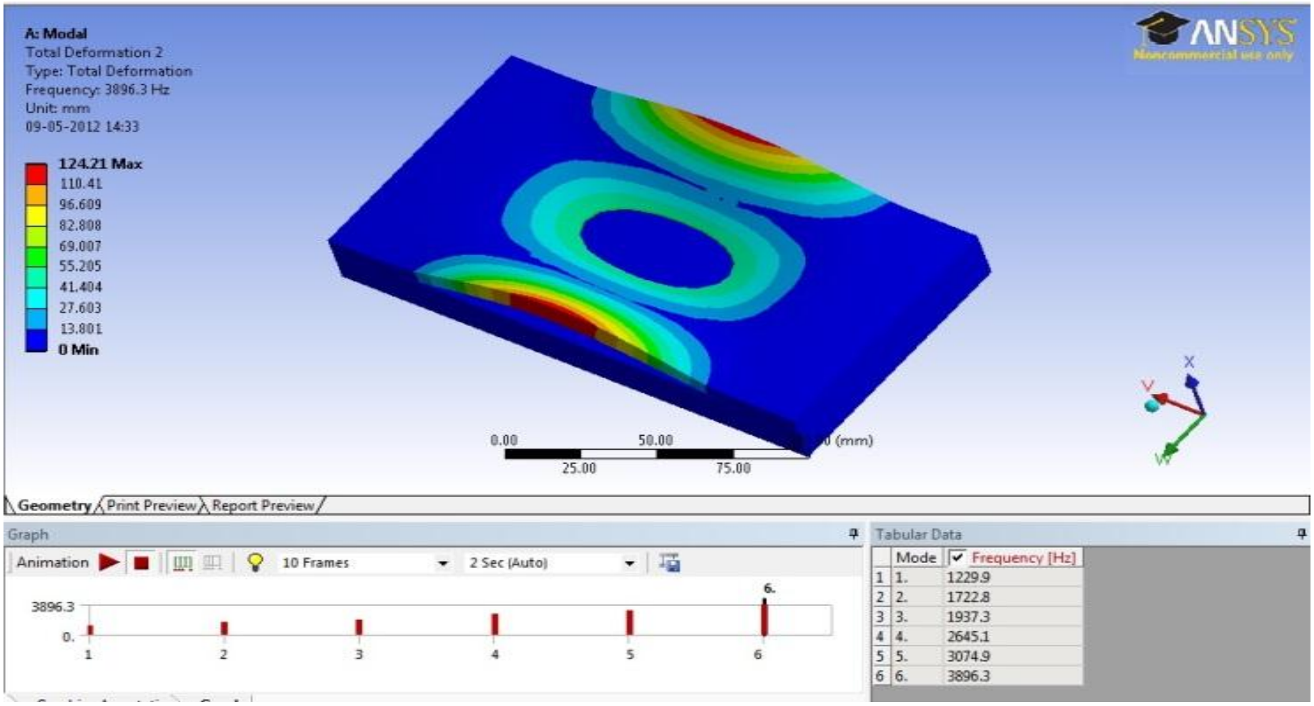
4th mode of vibration



5th mode of vibration

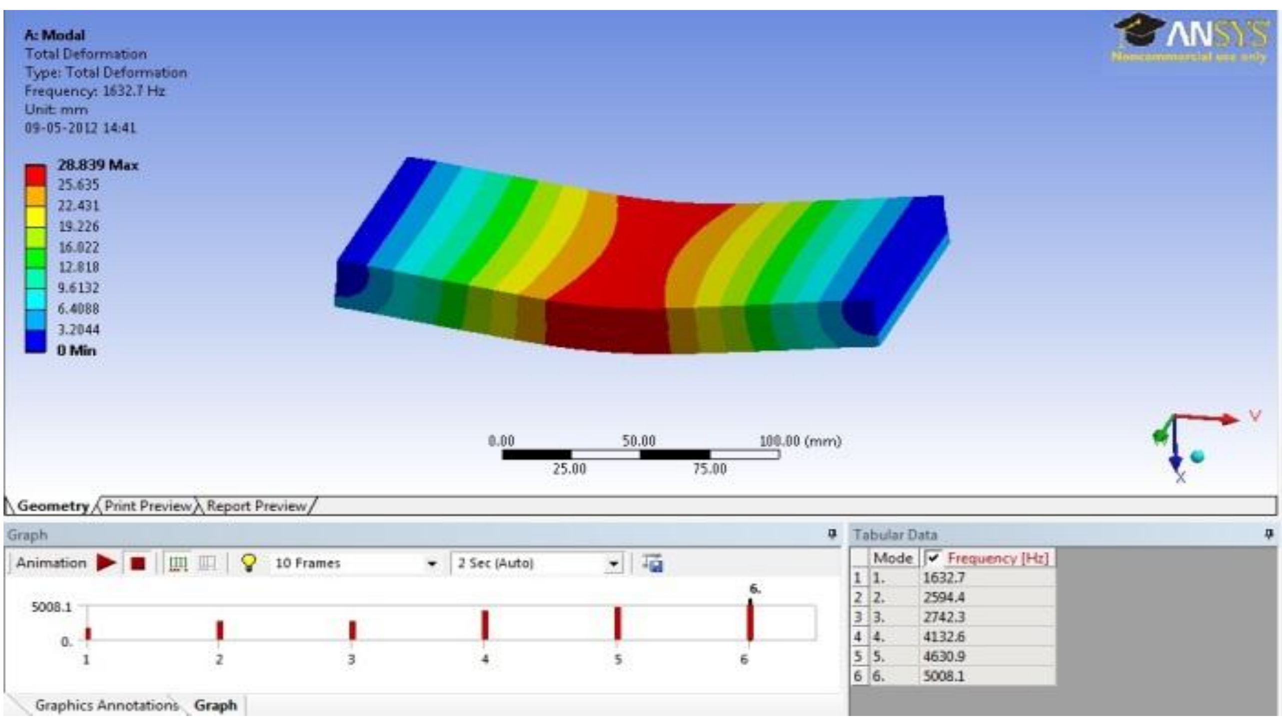


6^{th} mode of vibration

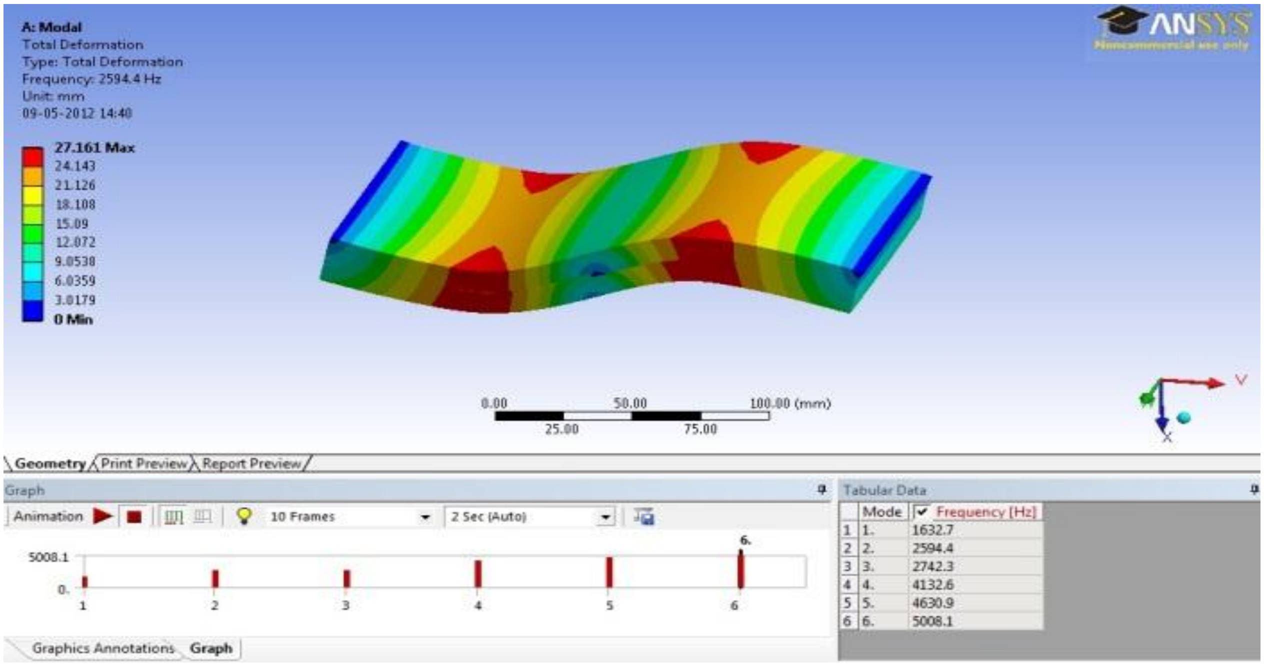


4^{th} case: plate 23, Dimensions: 200mm x 100mm x 20mm, (a/l) = 0.5, (t/h) = 0.5

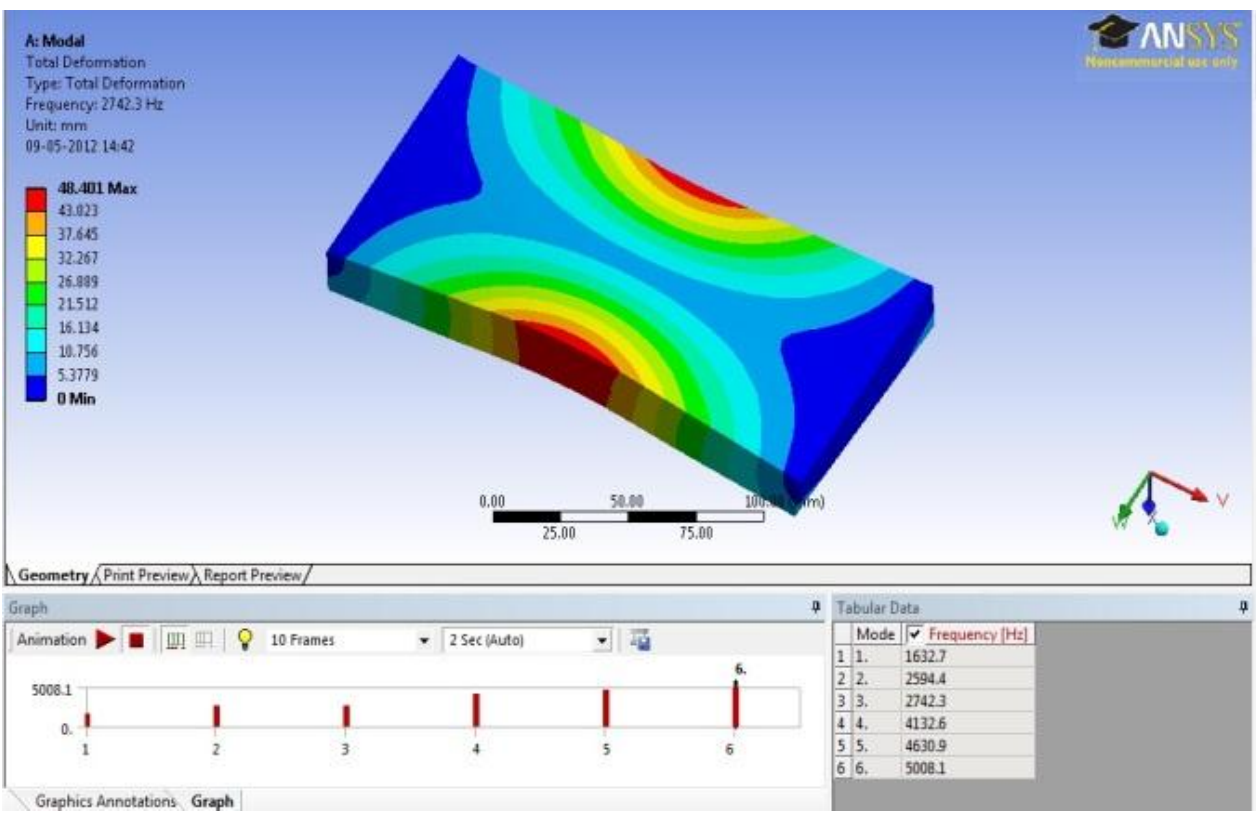
1^{st} mode of vibration



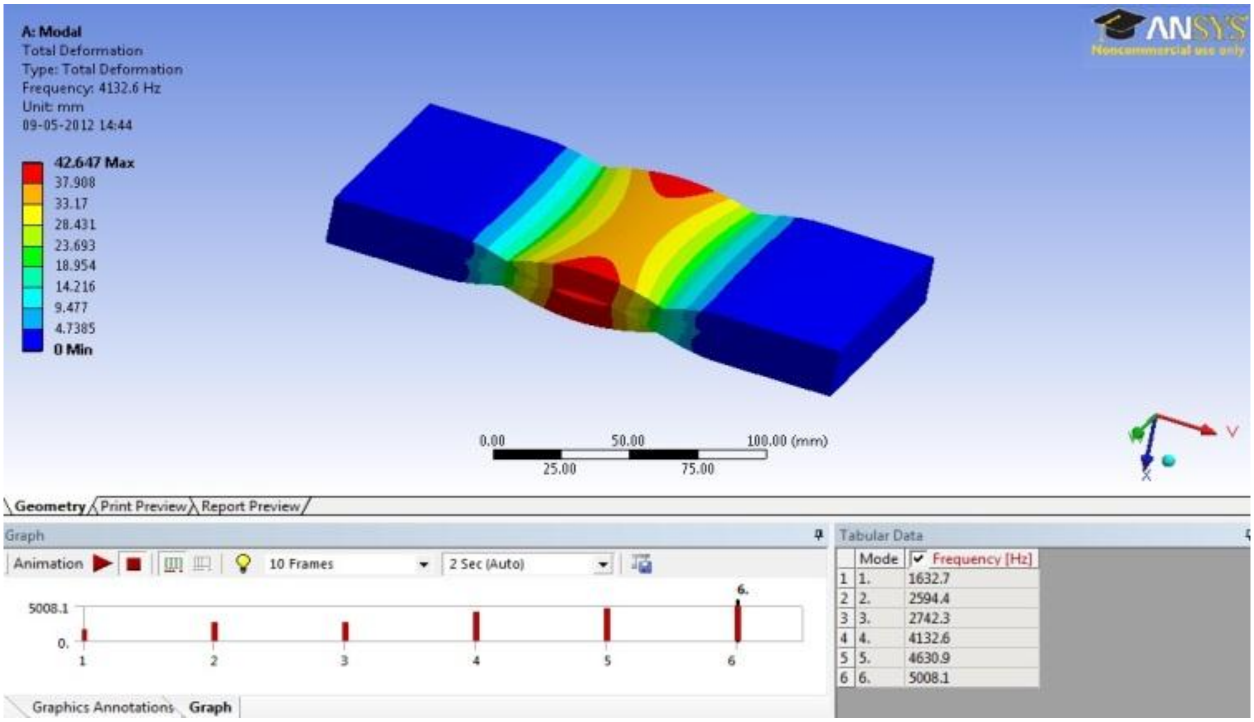
2nd mode of vibration



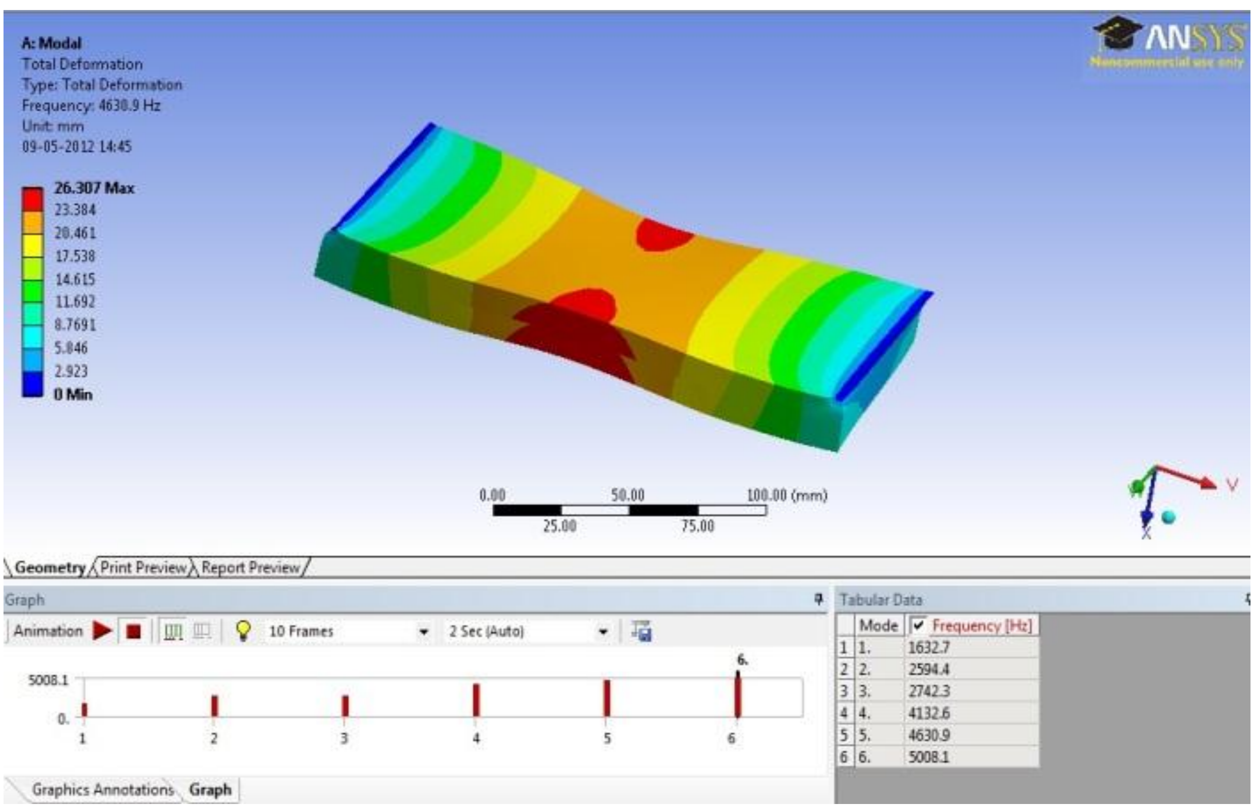
3rd mode of vibration



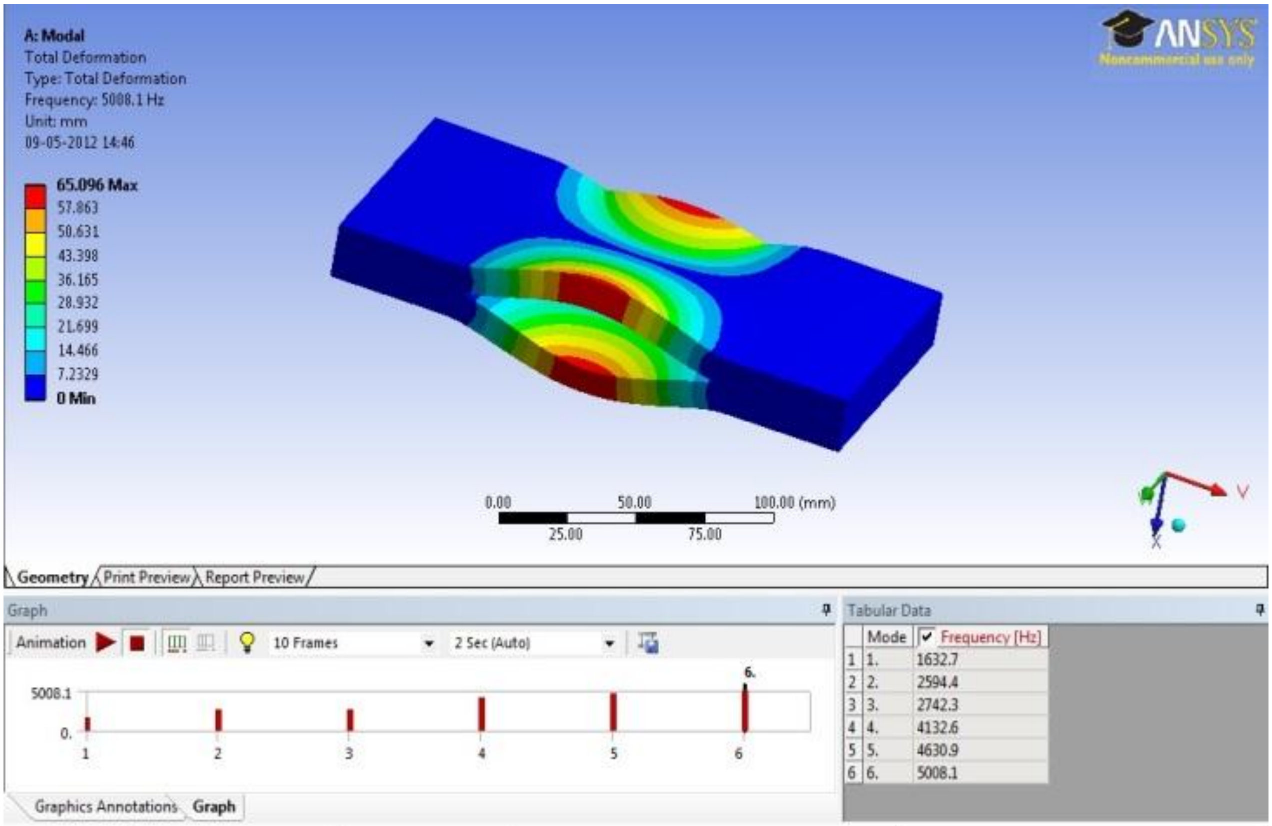
4th mode of vibration



5th mode of vibration



6^{th} mode of vibration



To verify our results we have compared them with case studied by Shiau and Zeng. [7]. In our formulation we used finite strip method to find the equations of the motion. From where, we found the frequency of the vibration of a simply supported delaminated plate. For the larger plates the Deformation patterns and results are found similar to those obtained for smaller plates.There are mainly three delamination variables which will affect the natural frequency of a delaminated plate. They are length of delamination (a), position of delamination (t) and mode of vibration, so we will discuss effect of each variable on the natural frequency.

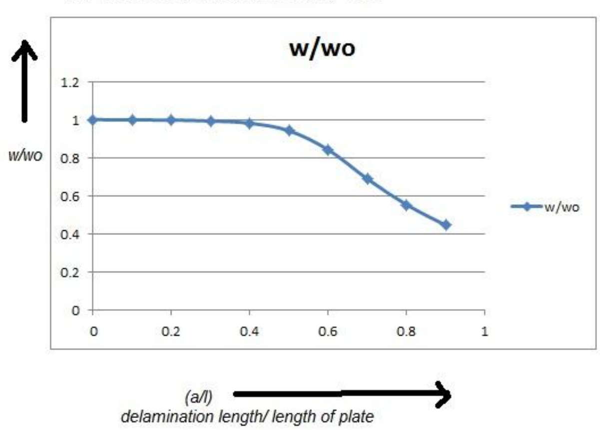
From the data and deformation patterns collected from Ansys13.0 analysis we will correlate these variables with frequency.

1. Effect of delamination length (a):

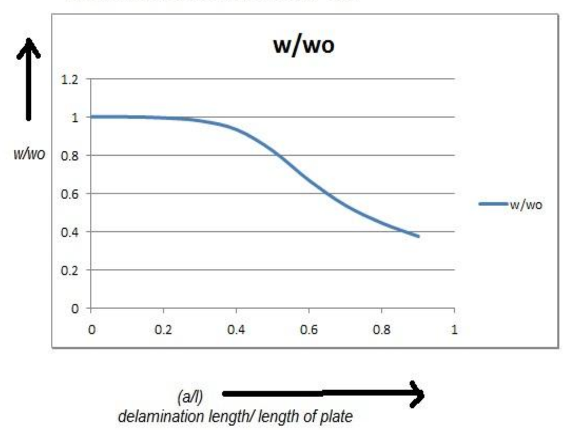
From past studies, it is observed that the natural frequency decreases with increase in delamination length. Natural frequency of the plate without delamination (plate 1) is *'fo'* and delamination length is '*a'.* From collected data we will draw a graph between *f/fo* and (*a*/*l*), keeping the other two variables constant.

1. For (t/h)= 0.25

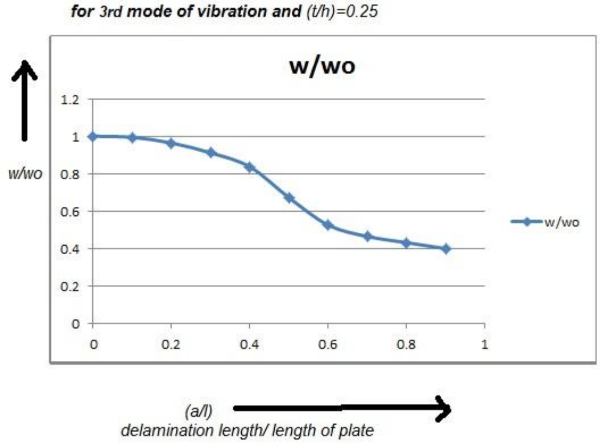
for 1st mode of vibration and (t/h)=0.25



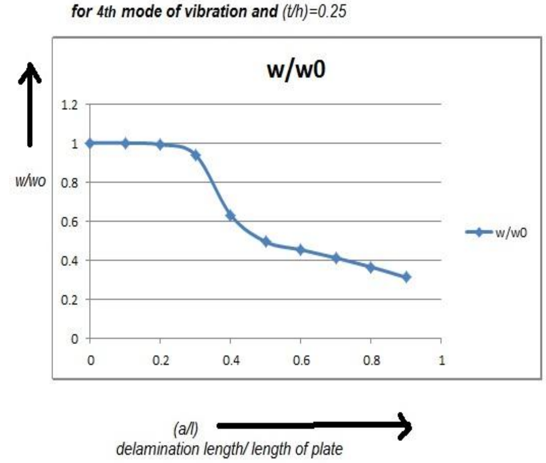
for 2nd mode of vibration and (t/h)=0.25

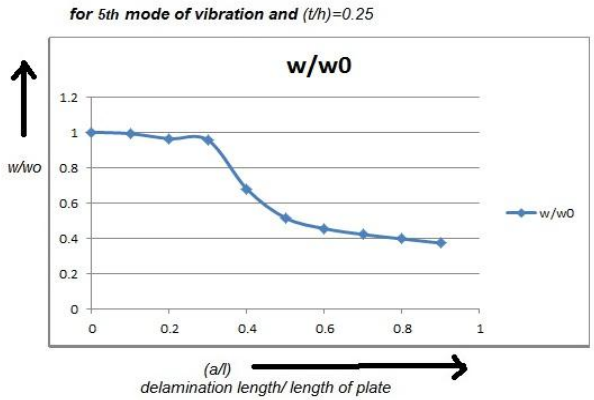


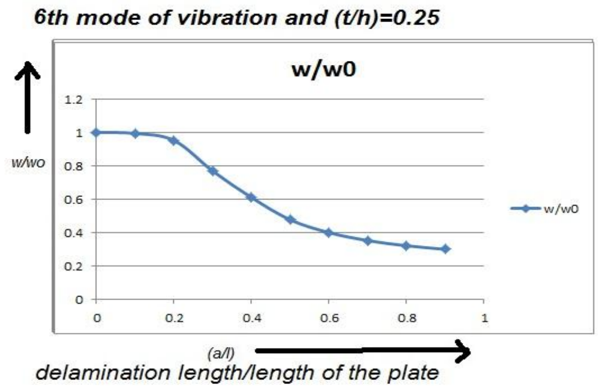
for 3rd mode of vibration and (t/h)=0.25



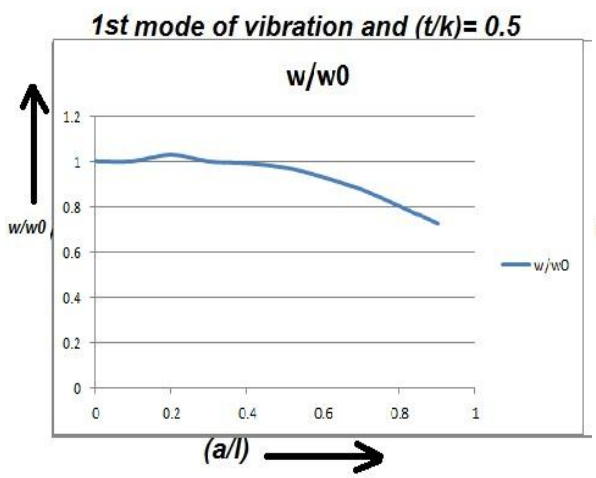
for 4th mode of vibration and (t/h)=0.25

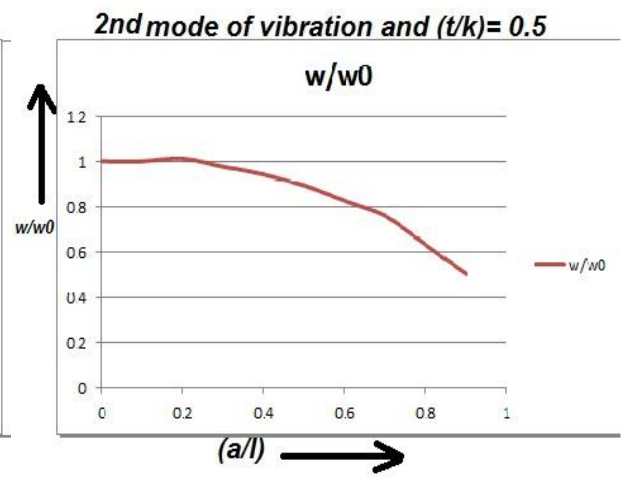


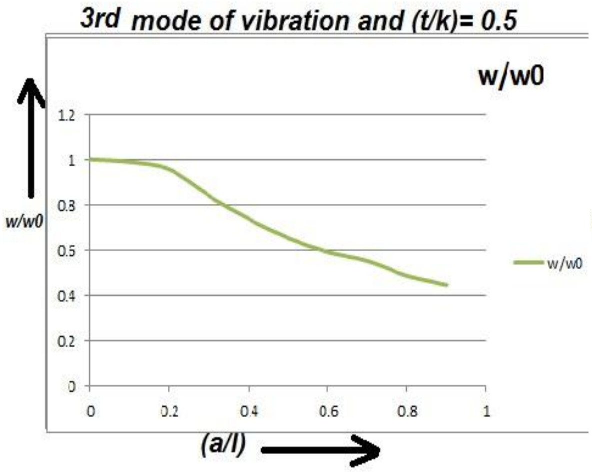

for 5th mode of vibration and (t/h)=0.25
w/w0
w/wo
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
w/w0
(a/l)
delamination length/ length of plate

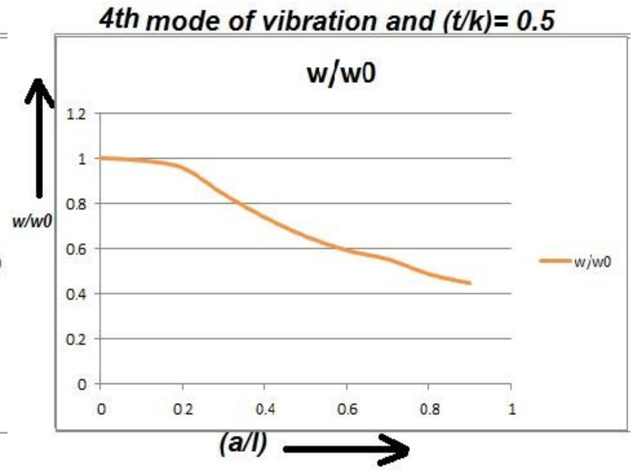

6th mode of vibration and (t/h)=0.25
w/w0
w/wo
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
w/w0
(a/l)
delamination length/length of the plate

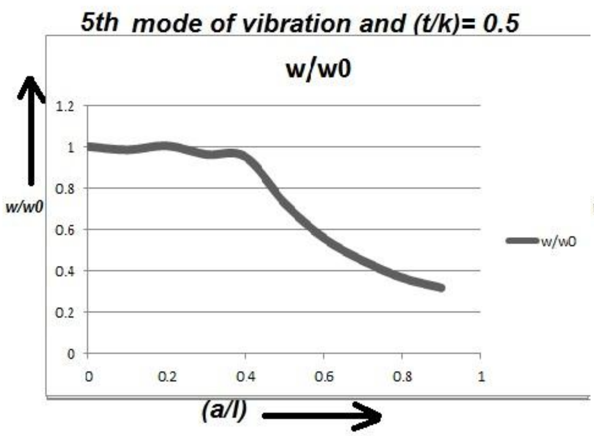
2. **For (t/h)= 0.5**

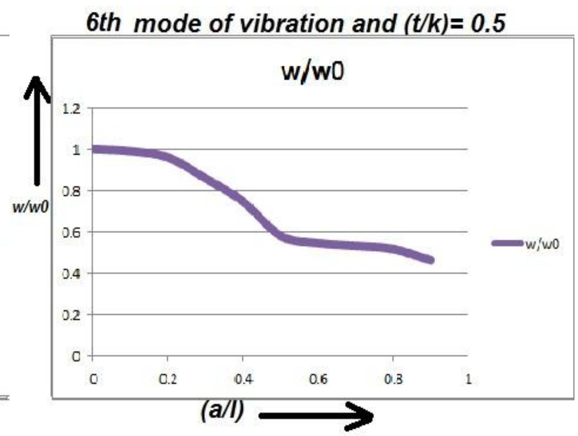

1st mode of vibration and (t/k)= 0.5
w/w0
w/w0
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
w/w0
(a/l)


2nd mode of vibration and (t/k)= 0.5
w/w0
w/w0
1.2
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1
w/w0
(a/l)









We studied the graphs for 12 different cases with varying (a/l) values and nature of graphs represented that the Frequency decreases with increase in delamination.Graphs between w/w_0 and (a/l) are in accordance with the results of the case study by Shiau and Zeng [7].

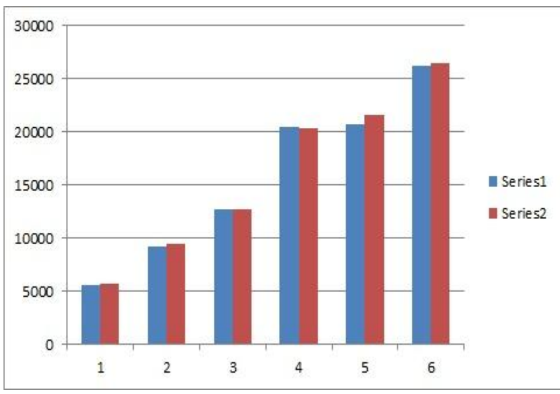
2. Effect of position of delamination (t):

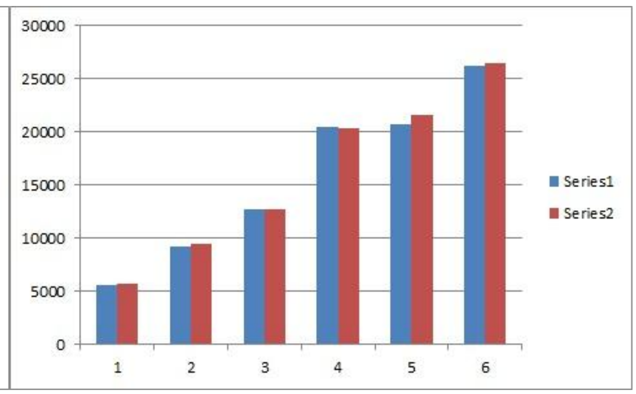
We examined the graph between w/w0 and (t/h) to understand the effect of t (distance of delaminated plane from upper plane of the plate) on the natural frequency of the plate. Mode of vibration and delamination lengths were taken as fixed values for each different case, while (t/k) was varied.

X axis =modes, Y axis= frequency

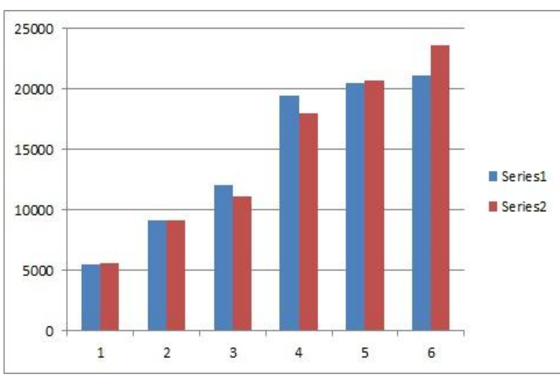
Blue= series 1 when (t/h)=0.25

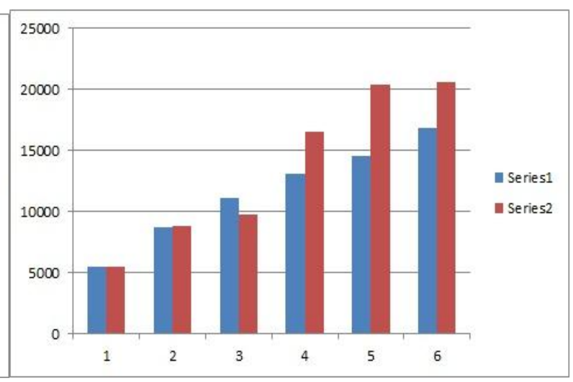
Red= series 2 when (t/h)= 0.5



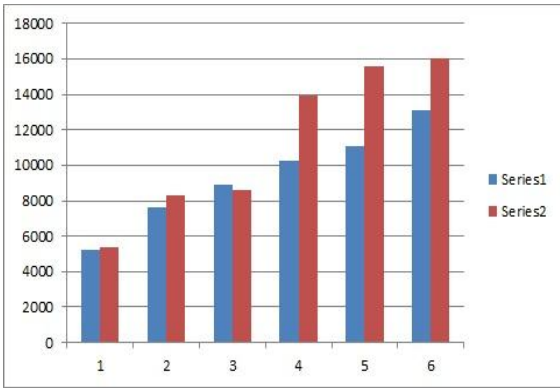


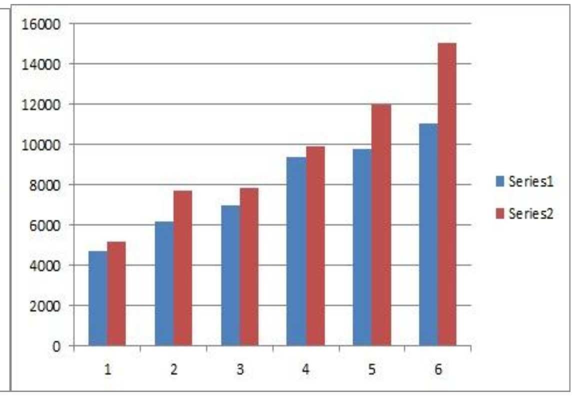
(a/l)=0.1 (a/l)=0.2



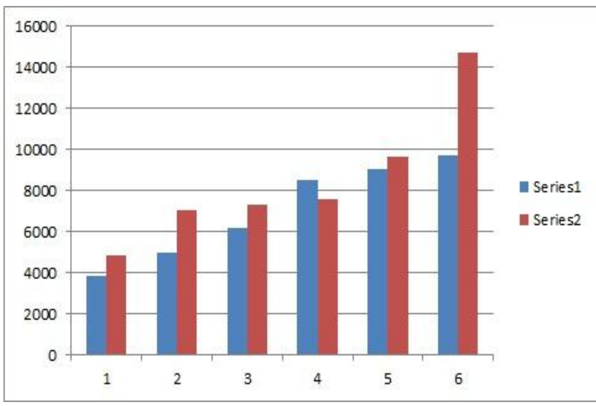


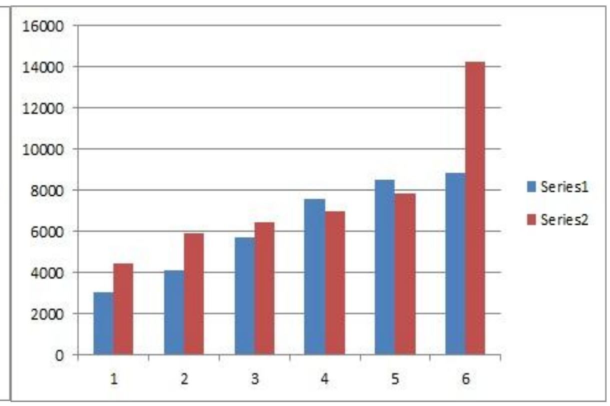
(a/l)=0.3 (a/l)=0.4



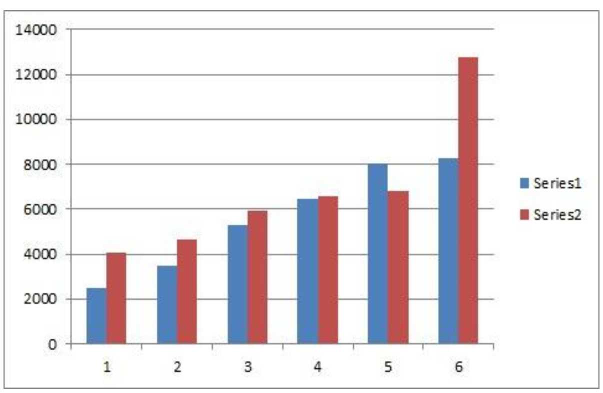


(a/l)=0.5 (a/l)=0.6





(a/l)=0.7 (a/l)=0.8



(a/l)= 0.9

Frequency is found to be increasing with increase in (t/k), this increment is more dominant at higher modes of vibration. For moderate modes of vibration, frequency decreased by some amount because in these conditions effect of delamination length is more dominant.

3. Mode of vibration:

From above data it is found that Vibration frequencies are high at higher nodes. When only mode of vibration increases, Frequency also increases {(a/l) and (t/h) are fixed}. The above results are compared with the results found by other case studies and the results are found satisfactory.

CHAPTER~6

6.Conclusion

Equations of motion are found using Finite element analysis and Ansys13.0 is used for the vibration analysis of the delaminated plates. Delamination variables are defined and values of frequencies are obtained using Ansys13.0 for different set of delamination variables. Using that data we have concluded that:

1. Frequency of a delaminated simply supported homogeneous plate decreases with increase in delamination length. Effect of delamination length is most dominant at moderate modes of vibration. At higher modes of vibration, effect of modes on frequency is dominant.

2. Frequency of a delaminated plate increases with increase in (t/h).

3. Frequency increases with increase in mode of vibration. For higher mode of vibration the frequency will be higher. Effect of this variable is dominant at higher modes of vibration.

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